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What Explains the Increasing Gap between Interest Rates and Return on Capital?

Accounting for Macro-Finance Trends: Cross Country Evidence

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Abstract

Over the past four decades interest rates have fallen remarkably reaching levels around zero in advanced economies. Meanwhile, the return on capital has maintained stable. Building on work by Farhi and Gourio (2018) we conduct an analysis of the increasing spread by augmenting a neoclassical growth model to incorporate risk and market power. We cover a wider set of developed economies and find that increasing risk and market power helps explain the widening gap between falling interest rates and stable returns on capital.

Keywords: Real interest rate, equity premium, return to capital, valuation ratios, price dividend, labour share, competition, markups, safe assets.

JEL classification: A22, E13, E44, G12, Y10.

*Dedicated to my father Magnus Petersen (1956–2019).

Contents

CONTRIBUTIONS:

Jens Brink – Sections 2, 3.2, 5, 6.2 and 6.3

Martin Nørgaard Petersen – Sections 3, 4, 6.1 and 6.4

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1 Introduction

Over the past four decades economists have observed a general decrease in real interest rates. At the same time, however, the average return to capital has remained stable. Together these facts have led to an increasing spread between the return on safe assets and that of risky assets, as illustrated by Figure 1. This raises the natural question —why are interest rates falling when the return on capital is not?

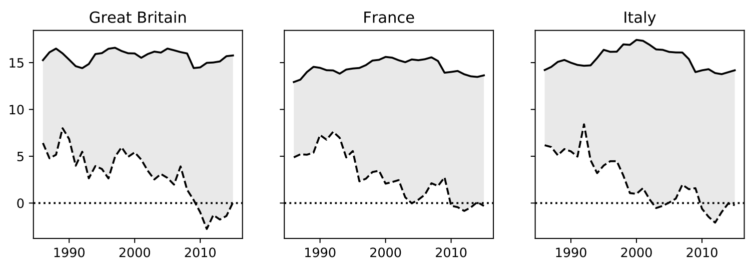
This is not a trivial question. Explaining the increasing spread has posed difficult for common macroeconomic modelling that does not incorporate risk[1]. The complexity of the question at hand has resulted in a one-sided focus on single trends, neglecting important evidence. Indeed, much of the current literature explains observed trends either by studying them in isolation, see e.g. Ferrero, Gross, and Neri (2017), or without targeting important macroeconomic variables, see e.g. Marx, Mojon, and Velde (2019).

A common explanation for explaining falling interest rates, such as seen in e.g. Carvalho, Ferrero, and Nechio (2016), is a demographic one; a general increase in savings supply from an aging population in advanced economies lead to lower interest rates. This hypothesis, however, would also imply a decrease in the average return to capital, which has generally not been the case, as evident from Figure 1.

A compelling explanation need to account for *both* the decrease in interest rates *and* the stability of the average return to capital, along with other important macro-finance trends. In this paper we seek to do exactly that. We adopt the model of Farhi and Gourio (2018), taking risk premia and market power into account, and apply it to a wide range of developed economies.

Our findings Focusing our analysis on Great Britain we find that the increasing wedge can be explained only by accompanying the increase in savings supply with increases in risk and firms' market power. In doing so we confirm results found by Farhi and Gourio (2018) for the US.

Figure 1: Real interest rate and average return to capital (1985-2015)



Notes: The dashed black line indicates the real interest rate. The solid black line marks the average return to capital. For time series definitions see Appendix, Section A. The grey area indicates the spread between the two rates. Units are percentages.

[1] In fact it has proved complex enough that even directors of central banks are left puzzled. *Per Callesen, director of the Danish Central Bank, at a gathering for the National Economic Association (NØF) of Denmark, cited with permission.*

This paper contributes by adding cross-country evidence to the analysis as well as making an easily editable online interface (build on Python) available, allowing for easy calibration of our model and for decomposing the effects of estimated parameter changes.

Programs and data All material, including data, programs and the most updated version of this paper can be found at `https://norgaardpetersen.dk/brink/`.

Programs for calibrating the model and decomposing as well as undertaking data cleaning and producing graphical outputs are written as Jupyter Notebooks. These iPython-files may be run from an online Binder, allowing programs to run through the users webbrowser. All can be found following the above URL.

Acknowledgements We would like to thank assistant professor Jeppe Druedahl for proficient supervision, helping out when stuck on math or code and answering an abundance of questions – often at odd hours. Further, we thank François Gourio and Emmanuel Farhi for inspiration and for making MATLAB files and other notes available to us.

2 Macroeconomic trends in Great Britain

The choice of which macroeconomic measures to focus on is motivated by the trends, we want our model to be able to account for. In addition to the wedge and commonly modelled relations, we include the investment to capital ratio to ensure consistency with actual capital accumulation, the price to dividend ratio to impute the return on equity, and growth of relative prices on investment in accordance with Greenwood, Hercowitz, and Krusell (2000). Lastly, we include the income share to capital to ensure reasonable estimates of markups.

This section describes the trends observed. We focus mainly on graphical evidence. In addition, for the case of Great Britain, Table 1 presents the simple mean of each time series in each of the two subsamples, we use in our estimation, as well as the change between periods.

These macroeconomic trends are generally found in most developed economies, and the analysis is thus relevant for a large range of countries. In this paper, we focus on Great Britain, chosen as the largest European country[2] with long time-series available[3]. In Section H and I of the appendix, we present the same time series for Denmark and a range of other developed countries, along with the results of our empirical analysis.

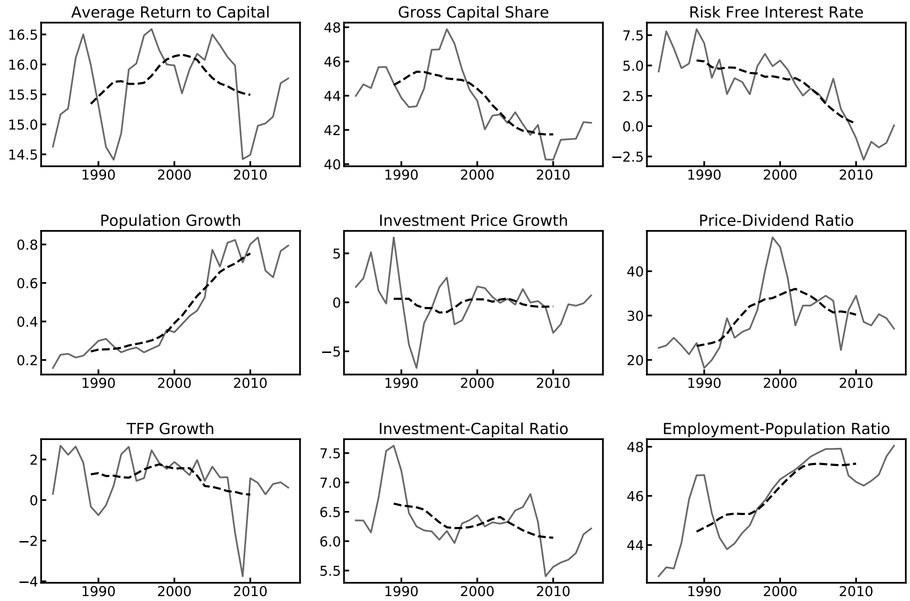
Our analysis spans over 30 years – from 1984 to 2015 – split equally at the millennium. This is partly a conscious choice, partly limited by data – by starting the analysis in 1984, we avoid most of the macroeconomic instability of the early 1980's; and the dataset we use is currently not updated with data later than 2015.

Table 1: Macroeconomic and Financial Trends in Great Britain

	Averages		
	1984 - 1999	2000 - 2015	Change
Average Return to Capital	15.631	15.630	−0.000
Gross Capital Share	45.147	42.064	−3.083
Risk Free Interest Rate	5.106	1.335	−3.771
Price-Dividend Ratio	26.679	31.775	5.097
Investment-Capital Ratio	6.491	6.143	−0.348
TFP Growth	1.363	0.662	−0.702
Investment Price Growth	0.231	−0.042	−0.272
Population Growth	0.255	0.652	0.396
Employment-Population Ratio	44.805	47.249	2.444

Notes: Change is absolute. See appendix, Section A, for data construction and sources.

Figure 2: Macroeconomic and Financial Trends in Great Britain, 1984 – 2015



The nine data series targeted in the estimation of the model, along with their respective 11-year centered moving average (dashed). All series are reported in percentages, except the price to dividend ratio.

2.1 Graphical analysis

The single most important trend for our analyses is the decrease in the real risk free interest rate[4], shown in the third panel of Figure 2. As evident, *the real risk free interest rate has decreased substantially* from around 5% to practically zero. The average return to capital, gross of depreciation, is shown in the first panel of Figure 2. The series shows business cycle-like fluctuation in the magnitude of one percentage point in either direction, but revolves around a relatively constant mean of 15.5%. Thus, over the sample period, *the average return to capital appears trendless.* In most other economies, the average return to capital increases slightly, c.f. appendix I. Combining the downward trending interest rate and the stable average return to capital introduces the increasing wedge, which motivates this paper.

Turning to the return from publicly traded companies, we use the price to dividend ratio —or reciprocal dividend yield— from the FTSE All-Share Index[5]. Using a composite index should be done cautiously as it introduces vulnerability

[2] Measured both in terms of gross domestic product and the ratio between exports and gross domestic product, important as our framework models a closed economy

[3] Of course, Germany would be the obvious choice, but the reunion in 1990 renders time-series inconsistent.

[4] Taken from the AMECO database. For construction of this and all other time series, we refer to appendix A. As noted by Caballero, Farhi, and Gourinchas (2017), the term risk-free is elusive as no asset is completely safe. This is also the case for our measure, though we continue to refer to it as risk free.

[5] We refer to the documentation in Jordà et al. (2019) for Dividend Yield measures for other economies.

to survivorship bias, given the fact that failing companies fall out of the index. Using this as a measure of return thus requires an assumption of continuously rebalanced portfolios – not far-fetched given the long time period. The price to dividend ratio is shown in the sixth panel of Figure 2, and while it spiked around the millennium, the ratio in the second sub-period has fallen to a level only slightly higher than in the first subsample. Given the large decrease in interest rates, and thus lower discount rates, *the price to dividend ratio has risen less than expected*[6].

The level of investment relative to the capital stock is shown in the eighth panel, and throughout the period, we find *slightly decreasing level of investment relative to capital*. Panel five shows the growth in relative prices of investment. This measure is remarkably stable around a zero mean, though with fluctuations, in particular before 2000.

The second panel of Figure 2 shows the gross capital share. As evident, the *capital share has fallen throughout the sample period*, with only a small increase since 2010. Note that this is contrary to most other developed economies. A possible explanation is that the decreasing capital share (and thus the rising wage share) is a result of the the combined growth in population and share of employed population evident in panel four and nine, respectively. Due to nominal wage rigidities in the economy, this could imply a medium-run increase in wage share. A different explanation is policy differences before and after the millennium. We do not speculate further, but return to the issue later.

Finally, we note that growth in total factor productivity has fallen slightly throughout the sample period, with a large drop around the financial crises of 2009, evident from the seventh panel of Figure 2.

[6]As evident from e.g. Gordons growth model, Gordon and Shapiro (1956), decreasing required return increases the value of stocks.

3 Theoretical framework

In the following section we present our theoretical framework. We entail a thorough description of the framework including derivations where necessary. Remaining derivations are enclosed in the appendix[7]. We apply Farhi and Gourio (2018) as frame of reference, however introduce slight changes in notation to avoid entanglement of terms.

The section is structured as follows: First we introduce our model from which we – in the next subsection – derive a *risky balanced growth path*. In the third subsection we then use our results from the risky balanced growth path to infer relevant model implications which we use in the section on identification of parameters. Our model transcends the standard neoclassical growth model by including risk, hence we present modelling of risk in a separate subsection, which is followed by a subsection on comparative statics discussing how changes in certain parameters affect the economy modelled.

3.1 The model

For our analysis we consider a dynastic model, where labour supply is assumed inelastic. The model resembles a standard neoclassical model with monopolistic competition but incorporates investment specific technological change and risk.

3.1.1 Utility function

We express utility using Epstein-Zin preferences[8], as they allow us to separate risk aversion from intertemporal elasticity of substitution, thus enabling the analysis of the role of risk:

$$V_t = \left((1-\beta) L_t c_{pc,t}^{1-\sigma} + \beta E_t [V_{t+1}^{1-\theta}]^{\frac{1-\sigma}{1-\theta}} \right)^{\frac{1}{1-\sigma}}, \tag{3.1}$$

where V_t is utility, L_t is the population size, $c_{pc,t}$ is consumption per capita in period t, σ is the inverse of the intertemporal elasticity of substitution (IES) and θ is the coefficient of relative risk aversion. Labour supply is assumed to be exogenous and given as $N_t = \bar{N} L_t$, where $\bar{N}$ is the employment-population ratio. We assume $\theta > \sigma > 0$, cf. Appendix, Section C.

The term $\beta E_t [V_{t+1}^{1-\theta}]^{\frac{1-\sigma}{1-\theta}}$ may be regarded as the *certainty equivalent* continuation value. E_t is the expectation in period t using all information available.

3.1.2 Output

At time t for firm i final output is produced with constant returns to scale using differentiated inputs

$$Y_t = \left(\int_0^1 y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{3.2}$$

[7]We are aware that this is unfortunately consistent with a modern trend in academic publishing of leaving mathematical derivations in the appendix, as criticised by McCloskey (1983). We hope the reader will excuse us, as we have merely sought to optimise model understanding subject to the imposed space constraint.

[8]We refer to appendix C for detailed notes on Epstein-Zin preferences.

where $\epsilon > 1$ is the elasticity of substitution and inputs are produced using a Cobb-Douglas production function given as:

$$y_{i,t} = Z_t k_{i,t}^{\alpha} (S_t n_{i,t})^{1-\alpha}, \tag{3.3}$$

where Z_t is an exogenous productivity trend, $n_{i,t}$ and $k_{i,t}$ are labour and capital inputs in firm i, and $1-\alpha$ is the (cost-based) labour share[9].

S_t is a stochastic productivity process, assumed to be martingale, meaning that the conditional expectation of productivity is equal to current level of productivity in any period given all previous realisations of the shock. We formulate this as:

$$S_{t+1} = S_t e^{\chi_{t+1}}, \tag{3.4}$$

where χ_{t+1} is assumed individually and identically distributed (*iid*).

3.1.3 Capital accumulation

Capital accumulation is given as the sum of capital after depreciation and investment. For the forthcoming derivation of the Euler equation (Section 3.1.5) we note that capital is our state variable, allowing us to describe the future behaviour of our model. We write:

$$k_{i,t+1} = ((1-\delta)k_{i,t} + x_{i,t}Q_t)e^{\psi_{t+1}}, \tag{3.5}$$

where we include a factor Q_t representing the current state of technology as in Greenwood, Hercowitz, and Krusell (2000). Q_t formalises how much capital can be purchased with one unit of output, hence changes in Q_t reflect investment-specific technological change[10]. Contrary to Greenwood, Hercowitz, and Krusell (2000, pp. 343–344) we do not differentiate between types of capital. As follows, the relative price of investment to consumption is $\frac{1}{Q_t}$.

Further, capital accumulation is subject to an aggregate capital quality shock ψ_{t+1}, which – like χ_{t+1} – is assumed *iid* with expected value 0[11].

3.1.4 Aggregated model

We assume that each firm i sets its prices $p_{i,t}$ and output $y_{i,t}$ so as to maximise profits given their demand curve, formally:

$$\max_{p_{it}, y_{it}} \{(p_{i,t} - mc_t) y_{i,t}\}$$
$$\text{subject to: } y_{i,t} = Y_t \left(\frac{p_{i,t}}{P_t}\right)^{-\varepsilon}$$

[9] Note that we differ between cost-based and income-based labour and capital shares. In section 3.3.4 we show that e.g. the income based shares are equal to cost-based shares when there is no market power, i.e. $\mu = 1$.

[10] This term covers both changes in costs of producing new equipment, and new equipment being more productive than older.

[11] Greenwood, Hercowitz, and Krusell (2000) argue that applying the invest specific technological change implies that GNP should not be adjusted to reflect quality improvements in new capital goods. One should bear this in mind if changing the distribution of ψ_{t+1} in a way that $E(e^{\psi_{t+1}}) \neq 1$.

Where P_t is the price-index which is set as numeraire $P_t = 1$. Maximising profits[12] implies that the optimal markup for each firm is equal to the inverse demand elasticity ε^{-1}:

$$\frac{1}{\varepsilon} = \frac{p_{i,t} - mc_t}{p_{i,t}},$$

meaning that all firms set the same price and in equilibrium we find that the firms are symmetric, ie. $n_{i,t} = N_t, y_{i,t} = Y_t, k_{i,t} = K_t$, and $p_{i,t} = P_t = 1$ and that marginal cost is:

$$mc_t = \frac{\varepsilon - 1}{\varepsilon} \equiv \mu$$

For wage w_t and rental rate of capital R_t we may regard marginal cost as the cost of expanding production using labour or capital, ie.:

$$mc_t = \frac{w_t}{MPL_t} = \frac{R_t}{MPK_t}, \tag{3.6}$$

where MPL_t and MPK_t are marginal products on capital and labour, respectively. This implies:

$$(1-\alpha)\frac{Y_t}{L_t} = \mu w_t \tag{3.7}$$

$$\alpha\frac{Y_t}{K_t} = \mu R_t. \tag{3.8}$$

As firms are symmetric, output and capital accumulation aggregates to:

$$Y_t = Z_t K_t^{\alpha}(S_t N_t)^{1-\alpha} \text{ and} \tag{3.9}$$

$$K_{t+1} = ((1-\delta)K_t - Q_t X_t)e^{\psi_{t+1}} \tag{3.10}$$

3.1.5 Euler Equation

The Euler equation expresses the optimal choice of investment, determined by the return to capital R^K_{t+1}:

$$1 = E_t[R^K_{t+1} M_{t+1}], \tag{3.11}$$

where M_{t+1} is the real stochastic discount factor.

Using equation (3.8) the return on capital (see derivation of Euler equation in Appendix, Section B), we find:

$$R^K_{t+1} = \left(R_{t+1} + (1-\delta)\frac{1}{Q_{t+1}}\right) Q_t e^{\chi_{t+1}}, \tag{3.12}$$

[12] First Order Condition:

$$\begin{aligned} & 0 = \frac{\partial}{\partial p_{i,t}}\left((p_{i,t} - mc_t)\left(\frac{p_{i,t}}{P_t}\right)^{-\varepsilon}\right) \\ \Leftrightarrow\quad & (1-\varepsilon)Y_t p_t^{\varepsilon} p_{i,t}^{-\varepsilon} = -\varepsilon mc_{i,t} Y_t P_t^{\varepsilon} p_{i,t}^{-(1+\varepsilon)} \\ \Leftrightarrow\quad & \frac{(1-\varepsilon)}{-\varepsilon} = \frac{mc_t}{p_{i,t}} \end{aligned}$$

which is a user cost expression including the rental rate of capital, depreciation and capital quality shocks. Note specifically that the relative price of capital goods affects the return on capital.

We have defined the stochastic discount factor as:

$$M_{t+1} \equiv \beta \left(\frac{c_{pc,t+1}}{c_{pc,t}} \right)^{-\sigma} \left(\frac{V_{pc,t+1}}{E_t[V_{pc,t+1}^{1-\theta}]^{\frac{1}{1-\theta}}} \right)^{\sigma-\theta}. \tag{3.13}$$

3.1.6 Resource Constraint

Finally, the resource constraint ensures that total output equals the sum of aggregate investment and consumption:

$$Y_t = X_t + C_t, \tag{3.14}$$

noting that $C_t = c_{pc,t}L_t$ and X_t is investment measured in consumption good units.

3.2 Risky balanced growth path

The model does not have a closed form solution and a non-stochastic solution does not enable the affect of macroeconomic risk. In a special case, however, we show that the model will follow a risky balanced growth path.

Firstly, as required for a balanced growth path, we assume that exogenous trends follow constant growth rates, identified as:

$$\frac{L_{t+1}}{L_t} = 1 + g_L, \frac{Z_{t+1}}{Z_t} = 1 + g_Z \text{ and } \frac{Q_{t+1}}{Q_t} = 1 + g_Q \text{ for } t \geq 0.$$

Secondly, we assume that the realised productivity shock χ always the capital quality shock ψ:

$$\chi_{t+1} = \psi_{t+1}, \tag{3.15}$$

meaning that whenever the economy is hit by a shock, capital adjusts simultaneously, eliminating the transition period after a shock.

3.2.1 Detrending the system

From the resource constraint, we find that output, consumption and investment must all follow the same trend T_t with a growth rate g_T. As level of capital is a result of investment and the relative price hereof, the trend in capital is accompanied by Q_t. Further, all variables are subject to the (equal) stochastic shock to productivity and accumulated capital S_t, which motivates us to define detrended variables as:

$$y_t \equiv \frac{Y_t}{S_t T_t}, x_t \equiv \frac{X_t}{S_t T_t}, c_t \equiv \frac{C_t}{S_t T_t}, v_t \equiv \frac{V_t}{S_t T_t} \text{ and } k_t \equiv \frac{K_t}{S_T T_t Q_t}. \tag{3.16}$$

Combining the production function from (3.9), the definition of labour force $N_t = \bar{N} L_t$ and detrended variables yields

$$\begin{aligned} Y_t &= Z_t K^\alpha (S_t N_t)^{1-\alpha} \\ Y_t &= Z_t (k_t S_t T_t Q_t)^\alpha (S_t \bar{N} L_t)^{1-\alpha} \\ Y_t &= S_t \underbrace{T_t^\alpha Z_t Q_t^\alpha L_t^{1-\alpha}}_{T_t} \underbrace{k_t^\alpha \bar{N}^{1-\alpha}}_{y_t}, \end{aligned}$$

why we find that:

$$\begin{aligned} y_t &= k_t^\alpha \bar{N}^{1-\alpha}, \text{ and} \\ T_t &= Z_t^{\frac{1}{1-\alpha}} Q_t^{\frac{\alpha}{1-\alpha}} L_t. \end{aligned}$$

From T_t, we derive the trend growth g_T, which we see is constant:

$$\boxed{\frac{T_{t+1}}{T_t} = 1 + g_T = (1+g_Z)^{\frac{1}{1-\alpha}} (1+g_Q)^{\frac{\alpha}{1-\alpha}} (1+g_L)}. \tag{3.17}$$

as above we box equations used for identification of parameters in section 4.2.

We then derive an expression for detrended investment x_t from capital accumulation, equation (3.10) and the detrended variables, eq. (3.16):

$$\begin{aligned} K_{t+1} &= ((1-\delta)K_t + Q_t X_t) e^{\chi_{t+1}} \qquad \text{(3.10 rev.)} \\ k_{t+1} S_{t+1} Q_{t+1} T_{t+1} &= ((1-\delta) S_t T_t Q_t k_t + S_t T_t Q_t x_t) e^{\chi_{t+1}} \end{aligned}$$

Using constant growth rates, as well as $S_{t+1} = S_t e^{\chi_{t+1}}$, we write:

$$\begin{aligned} k_{t+1} S_t e^{\chi_{t+1}} Q_t (1+g_Q) T_t (1+g_T) &= ((1-\delta) S_t T_t Q_t k_t + S_t T_t Q_t x_t) e^{\chi_{t+1}} \\ k_{t+1}(1+g_Q)(1+g_T) &= ((1-\delta)k_t + x_t), \end{aligned}$$

which we rearrange to obtain

$$x_t = k_{t+1}(1+g_Q)(1+g_T) - (1-\delta)k_t. \tag{3.18}$$

3.2.2 The system in equilibrium

In (general) equilibrium, detrended capital is constant, $k_{t+1} = k_t = k^\star$, thus

$$x^\star = k^\star((1+g_Q)(1+g_T) - (1-\delta)) \tag{3.19}$$

$$y^\star = (k^\star)^\alpha \bar{N}^{1-\alpha}. \tag{3.20}$$

This implies the following solutions to the system in equilibrium

$$\begin{aligned} K_t &= S_t T_t Q_t k^\star \\ Y_t &= S_t T_t y^\star \\ X_t &= S_t T_t x^\star, \text{ etc.,} \end{aligned} \tag{3.21}$$

where starred variables are constant on the risky balanced growth path. We note that X_t, Y_t grow at the same rates, however K_t grows faster for any positive value $g_Q > 0$, as capital becomes more productive, or cost of producing capital decreases.

Stochastic discount factor In equilibrium we may express the stochastic discount factor (see derivation in Appendix, Section D) from equation (3.13) as:

$$M_{t+1} = \beta \left(1 + g_{PC}\right)^{-\sigma} \left(e^{-\theta\chi_{t+1}}\right) \left(E_t \left[e^{(1-\theta)\chi_{t+1}}\right]\right)^{\frac{\theta-\sigma}{1-\theta}}, \tag{3.22}$$

where we have defined growth in output per capita as

$$1 + g_{PC} \equiv \frac{1 + g_T}{1 + g_L}.$$

Euler equation We recall the expression for the Euler equation from (3.11).

$$1 = E_t[R^K_{t+1} M_{t+1}] \tag{3.11 rev.}$$

Using our expression for M_{t+1} in equilibrium and the expression for R^K_{t+1} (equation (3.12)) we may expand terms and rearrange to arrive[13] at:

$$1 = E_t \left[\left(R_{t+1} Q_t + (1-\delta)\frac{Q_t}{Q_{t+1}}\right)\right] \beta^\star,$$

where we have defined the composite parameter:

$$\boxed{\beta^\star = \beta \left(1 + g_{PC}\right)^{-\sigma} \left(E_t \left[e^{(1-\theta)\chi_{t+1}}\right]\right)^{\frac{1-\sigma}{1-\theta}}}, \tag{3.23}$$

which encompasses the discount factor, growth in output per capita and risk. We then rearrange our expression of the composite parameter using that $R_t = \frac{\alpha}{\mu}\frac{Y_t}{K_t}$ to obtain (see full details in Appendix, Section E):

$$1 = \left(\frac{\alpha}{\mu}\left(\frac{k^\star}{\bar{N}}\right)^{\alpha-1} \frac{1}{1+g_Q} + \frac{1-\delta}{1+g_Q}\right)\beta^\star.$$

Consequently, the Euler Equation in equilibrium reads

$$\frac{1}{\beta^\star} = \left(\frac{\alpha}{\mu}\left(\frac{k^\star}{\bar{N}}\right)^{\alpha-1} \frac{1}{1+g_Q} + \frac{1-\delta}{1+g_Q}\right) \tag{3.24}$$

We denote $\beta^\star$ the *effective* discount factor, and define the relevant rate of return in net terms as

$$r^\star = \frac{1}{\beta^\star} - 1 \tag{3.25}$$

3.3 Model implications

This section derives implications from the model in equilibrium used for identification as well as relations useful for interpreting parameter effects.

[13] See full derivation in Appendix, Section E

3.3.1 User cost of capital

We may rearrange equation (3.24) to yield:

$$\left(\frac{1+g_Q}{\beta^\star} - (1-\delta)\right) = \frac{\alpha}{\mu}\left(\frac{k^\star}{\bar{N}}\right)^{\alpha-1}$$
$$\frac{1}{\beta^\star} - 1 + \delta + \frac{g_Q}{\beta^\star} = \frac{\alpha}{\mu}\left(\frac{k^\star}{\bar{N}}\right)^{\alpha-1}$$

Which we, using $\frac{g_Q}{\beta^\star} \approx g_Q$, may approximate as:

$$r^\star + \delta + g_Q = \frac{\alpha}{\mu}\left(\frac{k^\star}{\bar{N}}\right)^{\alpha-1}, \tag{3.26}$$

that is, the user cost of capital equals the marginal revenue. The equation indicates that higher market power or higher required *risky* return leads to a lower desired ratio of capital to labour. We later use this equation to find the average return to capital in section 3.3.5.

3.3.2 Capital-output ratio

The current capital stock at current costs is denoted K_t/Q_t. Using the detrended values for K_t and Y_t from eq. (3.21), and $y_t = k^\alpha \bar{N}^{1-\alpha}$ from eq. (3.20), we obtain

$$\frac{K_t/Q_t}{Y_t} = \frac{k_t^\star S_t T_t Q_t/Q_t}{y_t^\star S_t T_t} = \frac{k^\star}{y^\star} = \frac{k^\star}{{k^\star}^\alpha \bar{N}^{1-\alpha}}$$

Rearranging the equation for user cost of capital, eq. (3.26), and plugging the above expression in yields

$$\frac{K_t/Q_t}{Y_t} = \frac{\alpha}{\mu}\frac{1}{r^\star + \delta + g_Q}, \tag{3.27}$$

which shows that desired capital-output ratio is increasing in α, the output elasticity of capital, but decreasing in both markups, interest rate and depreciation.

3.3.3 Investment-capital ratio

The investment-capital ratio may be calculated using detrended investment, equation (3.18) as:

$$\boxed{\frac{X_t}{K_t/Q_t} = \frac{x_t}{k_t} = (1+g_Q)(1+g_T) - (1-\delta)} \tag{3.28}$$

which, using $g_Q g_T \approx 0$, approximates to :

$$\frac{X_t}{K_t/Q_t} \approx g_Q + g_T + \delta, \tag{3.29}$$

showing that increases in economic and physical depreciation as well as higher output growth leads to a higher investment-to-capital ratio.

3.3.4 Capital Income Share

From the firms optimisation problem, equation (3.7) we note that:

$$s_L = \frac{L_t w_t}{Y_t} = \frac{1-\alpha}{\mu}$$

from which we find that the income share to capital is

$$\boxed{s_K = 1 - s_L = \frac{\mu+\alpha-1}{\mu}}, \tag{3.30}$$

which can decomposed into a pure profit share, rewarding capital owners for monopoly rents, and a share corresponding to the rental payments to capital:

$$s_K = s_\pi + s_c = \frac{\mu-1}{\mu} + \frac{\alpha}{\mu}. \tag{3.31}$$

As noted this simplifies to the cost-based capital share for $\mu = 1$, i.e. if firms have no market power.

3.3.5 Rates of return

Risk free rate Although the risk free rate is not traded, we may price it as the inverse of the stochastic discount factor:

$$\begin{aligned} RF &= \frac{1}{E_t[M_{t+1}]} = \frac{1}{E_t\left[\beta\left(1+g_{PC}\right)^{-\sigma}\left(e^{-\theta\chi_{t+1}}\right)\left(E_t\left[e^{(1-\theta)\chi_{t+1}}\right]\right)^{\frac{1-\sigma}{1-\theta}-1}\right]} \\ &= \frac{1}{E_t\left[\beta^\star\left(e^{-\theta\chi_{t+1}}\right)\right]E_t\left[e^{(1-\theta)\chi_{t+1}}\right]^{-1}} = \frac{E_t\left[e^{(1-\theta)\chi_{t+1}}\right]}{\beta^\star E_t\left[e^{-\theta\chi_{t+1}}\right]}. \end{aligned}$$

In net terms this yields:

$$\boxed{rf = RF - 1 = \frac{E_t\left[e^{(1-\theta)\chi_{t+1}}\right]}{\beta^\star E_t\left[e^{-\theta\chi_{t+1}}\right]} - 1} \tag{3.32}$$

Average return to capital We infer the average return to capital net of depreciation in the following way, remembering that $s_K = s_C + s_\Pi$:

$$\frac{s_K Y_t}{K_t/Q_t} = \frac{s_K y^\star}{k^\star} = s_K \frac{(k^\star)^\alpha \bar{N}^{1-\alpha}}{k^\star} = \frac{\mu+\alpha-1}{\mu}\left(\frac{k^\star}{\bar{N}}\right)^{\alpha-1}$$

We use the expression for user cost of capital derived in equation (3.26), yielding

$$\frac{s_K Y_t}{K_t/Q_t} = \frac{\mu+\alpha-1}{\mu}\frac{\mu}{\alpha}\left(r^\star + \delta + \frac{g_Q}{\beta^\star}\right), \tag{3.33}$$

which[14] gives us:

$$\boxed{\frac{s_K Y_t}{K_t/Q_t} = \frac{\mu+\alpha-1}{\alpha}\left(r^\star + \delta + \frac{g_Q}{\beta^\star}\right)}. \tag{3.34}$$

[14]Note, that in (3.26) we approximated $\frac{g_Q}{\beta^\star} \approx g_Q$ to show that (approximate) user cost equals marginal revenue. However for precision of results when calibrating the model we use the precise expression.

We may decompose the spread between the average return to capital and the risk free rate as expression as:

$$\frac{s_K Y_t}{K_t/Q_t} - r_f \approx (\delta + g_Q) + \frac{\mu - 1}{\alpha}(r^\star + \delta + g_Q) + (r^\star - r_f) \tag{3.35}$$

which highlight that average return to capital exceeds the risk free rate for three reasons. Firstly, because of physical and economic depreciation $(\delta + g_Q)$, and secondly, because it is *risky* $(r^\star - r_f)$. Thirdly we note that the spread is further increased through a profit rents share $(\frac{\mu-1}{\mu})$ of user cost of capital.

3.3.6 Gordon growth formula

Let P_t denote the price of equity at time t and D_t the dividend payments. Then the (relative) return on equity is given as:

$$R^E_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} \tag{3.36}$$

Which we may rearrange[15] and insert in our restated Euler equation:

$$1 = E_t\left[M_{t+1} R^E_{t+1}\right]$$

Inserting our rearranged expression for the return on equity (3.36) (see algebra in Appendix, Section F) yields:

$$\begin{aligned} 1 &= E_t\left[M_{t+1}\left(\frac{P_{t+1}}{D_{t+1}} + 1\right)\frac{D_{t+1}}{D_t}\frac{D_t}{P_t}\right] \\ \frac{P_t}{D_t} &= E_t\left[M_{t+1}\left(\frac{P_{t+1}}{D_{t+1}} + 1\right)\frac{D_{t+1}}{D_t}\right] \end{aligned}$$

The price dividend ratio is assumed constant along the risky balanced growth path. Further, using that $E(e^{\chi_{t+1}}) = 1$ and equating dividends and profits we may rewrite the expression in terms of the composite parameter $\beta^\star$:

$$\frac{P^\star}{D^\star} = \left(\frac{P^\star}{D^\star} + 1\right)\beta^\star(1 + g_T)$$

Which can be rearranged as:

$$\boxed{\frac{P^\star}{D^\star} = \frac{\beta^\star(1 + g_T)}{1 - \beta^\star(1 + g_T)}} \tag{3.37}$$

As $1 + r^\star = \frac{1}{\beta^\star}$ we recover the Gordon growth formula as:

$$\begin{aligned} \frac{P^\star}{D^\star} &= \frac{\beta^\star(1 + r^\star)(1 + g_T)}{(1 + r^\star) - \beta^\star(1 + r^\star)(1 + g_T)} \\ &= \frac{(1 + g_T)}{(1 + r^\star) - (1 + g_T)} \\ &= \frac{1 + g_T}{r^\star - g_T}. \end{aligned} \tag{3.38}$$

[15] See full derivation of Gordon Growth Formula in Appendix, Section F.

On equating returns When restating the Euler equation above, we implicitly match the return on equity to that of capital, relying on the traditional neoclassical framework, in which the no-arbitrage condition is assumed to hold. This is not a straight forward assumption, as pointed out by e.g. Gomme, Ravikumar, and Rupert (2011), whose findings we discuss later in Section 6.

3.4 Modelling risk

We model risk as the probability of an economic shock. In particular, we model a macroeconomic shock χ_{t+1} to follow a three-point distribution:

$$\begin{aligned} \chi_{t+1} &= 0 \text{ with probability } 1-2p \\ \chi_{t+1} &= \ln 1 - b \text{ with probability } p \\ \chi_{t+1} &= \ln 1 + b \text{ with probability } p, \end{aligned} \tag{3.39}$$

implying that $E(e^{\chi_{t+1}}) = 1$ (see Appendix, Section G). In other words, the economy is hit by a 'disaster' scenario of a permanent reduction of output and consumption by a factor $1-b$ (where $b \geq 0$) with probability p. And equally with probability p, output and consumption increases by a factor $1+b$; we term this event the 'windfall' scenario. As we return to in Section 6, the modelling of risk is not self-evident and may be subject to further scrutiny[16].

In our applications we assume that the size of macroeconomic shock is $b = 0.15$. Based hereupon we may estimate the probability of a shock p using the equation for the risk free rate (3.32).

3.5 Comparative statics

This section discusses the effects of certain parameter changes on key measures of our model. We focus on parameters specific to our model and that have relevance for interpreting our results in the following section.

3.5.1 Effects of risk

A main feature of our model is the incorporation of risk. Consider a change in the probability of a shock ('disaster' or 'windfall', respectively). From equation (3.23), which identifies $\beta^\star$, we note that the term

$$E_t[e^{(1-\theta)\chi_{t+1}}] = (1-2p)e^0 + p \cdot e^{(1-\theta)\ln(1-b)} + p \cdot e^{(1-\theta)\ln(1+b)}$$

varies positively with p for $\theta > 0$[17]. Consequently, the term

$$E_t[e^{(1-\theta)\chi_{t+1}}]^{\frac{1-\sigma}{1-\theta}}$$

moves inversely with p if and only if $\sigma < 1$, i.e. IES is larger than unity (as we assume in our application). Thus, other factors constant, an increase in the probability of a shock implies a decrease in the effective discount factor $\beta^\star$.

In turn, this means that the required risky return $r^\star = \frac{1}{\beta^\star} - 1$ increases, implying an increase in the spread between the risk free rate and the average

[16] Specifically, one may argue that a two-point distribution consisting of the first and second event would be a more 'realistic'. This would, however, incur mean effects as $E(e^{\chi_{t+1}}) \neq 1$.

[17] For simplicity we assume $\theta > 1$. See also Appendix, Section C.

return to capital as seen by equation (3.35). Here we note that the effect on the spread stems from increased risk, as the risk premium rises. Additionally, the spread increases due to increased profit rents, as these provide an additional share of user cost of capital (which entails $r^\star$).

Assuming that μ is relatively close to one, we argue that the spread depends positively on p, independent of the intertemporal elasticity of substitution[18].

Further, from equations (3.26) and (3.27) we note that an increase in $r^\star$ leads to a lower desired capital-labour ratio and a decrease in the capital-output ratio, as higher risk discourages investment. It follows that an increase in the probability of a shock leads to a decrease in capital accumulation and thus an increase in average return to capital following from diminishing average returns[19].

Moreover, the increase in $r^\star$ leads to a decrease in the price dividend ratio as follows from the Gordon growth formula (3.37) if the intertemporal elasticity of substitution is high (i.e. $\sigma < 1$). In short, higher risk leads to lower valuations. Conversely, if the intertemporal elasticity of substitution is low (i.e. $\sigma > 1$), agents expect a lower return on capital from an increase in the probability of a shock, resulting in higher valuations.

3.5.2 Effects of savings supply

The effect of savings supply enters the model through β, the 'impatience' of agents, which affects the composite parameter $\beta^\star$ as expressed in equation (3.23), and hence acts through many of the same channels as an increase in the probability of a shock would: An increase in savings supply increases the effective discount factor, decreasing the required return on risky capital $r^\star$, implying a higher price dividend ratio, higher investment-capital ratio and lower average return to capital.

The primary difference from above is that the risk free rate, which is affected both through $\beta^\star$ and through the distribution of risk, see equation (3.32). In the case of an increase in savings supply, the effect is one-sided, putting downward pressure on the risk-free rate.

As seen from footnote 18, the spread $r^\star - r_f$ is approximately independent of β, as it enters both rates, and thus the average return to capital is only affected through profit rents. Hence, the spread between risk free rate and average return *decreases* slightly for an increase in savings supply.

[18] We note that we may rewrite the equation (3.32) for the risk free rate using log-approximations as

$$\begin{aligned} rf &\approx r^\star + \ln E_t\left[e^{(1-\theta)\chi_{t+1}}\right] - \ln E_t\left[e^{(-\theta)\chi_{t+1}}\right] \\ \Leftrightarrow r^\star - rf &\approx \ln E_t\left[e^{(-\theta)\chi_{t+1}}\right] - \ln E_t\left[e^{(1-\theta)\chi_{t+1}}\right] \end{aligned}$$

The derivative is positive wrt. $p \in [0,1]$. Hence, the spread $r^\star - r_f$ is positively correlated with p and independent of the intertemporal elasticity of substitution.

[19] Diminishing marginal returns implies diminishing average returns. Further note that the average return to capital equals the marginal product of capital under perfect competition and constant returns to scale. The reader may confirm this by setting $\mu = 1$ in the expression for average return to capital from equation (3.34) and equate this with the marginal product wrt. capital in equation (3.9).

3.5.3 Effects of other parameters

Increased depreciation We may consider an increase in the rate of depreciation of physical capital δ. This directly increases the user cost of capital increases, why the desired capital to labour and capital to output ratios decrease, see equations (3.26) and (3.27).

As the user cost of capital increases, an increase in depreciation implies a decrease in the investment to capital ratio, consistent with a lower capital accumulation, see eq. (3.29). Gross average return to capital increases with depreciation, again as a result of increasing user cost of capital. As does the spread to the risk-free rate, since the risk-free rate is priced independently of depreciation, see (3.32).

Note that we have not differentiated between structures and equipment as in Greenwood, Hercowitz, and Krusell (2000). As they show for the US, and as can be confirmed by casual observation, depreciation of equipment (such as computer, telecommunication, etc.) exceeds that of structures. Hence, an increase in δ might not only arise from general increases in depreciation, but also from a shift in the distribution of structures and equipment.

Effect of market power An increase in market power in particular affects income distribution in the economy. As firms optimize profits – not production – higher markups lead to higher prices. This increases profits but lowers the demanded quantity, why total production falls. Lower output implies a lower amount of inputs, why the income share to labour and capital decreases, though the income share to profits rises, as previously noted.

An increase in market power also affects the capital-labour ratio negatively as firms have decreasing incentive to build capital capacity.

Average return to capital is increasing in markups, however, as the profit rents depend positively on markups – with the logic that higher prices lead to higher profitability. The spread to the risk-free rate thus increases, since no change to the risk free asset or the pricing hereof occurs, see equation (3.35).

4 Methodological framework

This section first describes our applied methodology. Second it describes our identification of the parameters from moments. And finally we describe the method of decomposition used to ascribe the total change in moments to individual contributions from parameters.

4.1 Methodology

We use method of moments estimation to estimate nine parameters from nine moment conditions. The exact identification approach combined with a partly recursive identification eases the understanding of moments' effect on parameters. We compare estimates for the model from two periods; 1984–1999 and 2000–2015, in all cases fitting the parameters derived from the risky balanced growth path to the data moments.

We present the target moments and parameters to be estimated in the following table:

Table 2: List of target moments and estimated parameters

	Target moments		**Parameters**
M1	Population Growth	**P1**	Population Growth, g_L
M2	Investment Prices Growth	**P2**	Invt. Specific Progress, g_Q
M3	Employment-Population Ratio	**P3**	Labour Supply Parameter, $\bar{N}$
M4	TFP Growth	**P4**	TFP Growth, g_Z
M5	Gross Capital Share, s_K	**P5**	Cobb-Douglas Parameter, α
M6	Investment-Capital Ratio, $\frac{I}{K/Q}$	**P6**	Markup, μ
M7	Price-Dividend Ratio, $\frac{P}{D}$	**P7**	Depreciation rate of capital, δ
M8	Avg. Return to Capital, $\frac{s_K Y}{K/Q}$	**P8**	Risk parameter, p
M9	Risk Free Interest Rate, rf	**P9**	Discount factor, β

Notes: For target moments we use the following shorthand we use GrowthPop, PriceInvt, EmpPop, TFPgrowth, XK, PD, CapShare, AvgRet, rf.

4.2 Identification

In this section we describe the identification of the above parameters and present all moment conditions. Calibration-files written in iPython allowing for easy calibration for each country may be downloaded from `https://norgaardpetersen.dk/brink/`. Link to an online Binder, allowing the reader to run our programs in a webbrowser, can also be found here.

4.2.1 Moment Equivalents

Parameters g_L, g_Q and $\bar{N}$ are set to match their moment equivalents directly. Thus, moments conditions are:

$$E[\mathsf{GrowthPop}_t] = g_L \qquad E[\mathsf{PriceInvt}_t] = g_Q \qquad E[\mathsf{EmpPop}_t] = \bar{N} \tag{4.1}$$

4.2.2 Equation system

We note that the total factor productivity growth measured in the model is revenue based and not cost-based, meaning that the share of labour is $s_L = \frac{1-\alpha}{\mu}$ and not $1-\alpha$. Consequently, the total factor productivity growth as measured in our model is:

$$\begin{aligned} E[\mathsf{TFPgrowth}_t] &= g_T - s_L g_L - (1-s_L) g_K \\ &= g_T - s_L g_L - (1-s_L)(g_T + g_Q) \end{aligned} \tag{4.2}$$

This implies[20] that we need α to identify g_T, hence our estimation is not fully recursive. Using (4.2) and equation (3.17) leads to identification of g_T:

$$1 + g_T = (1+g_Z)^{\frac{1}{1-\alpha}} (1+g_Q)^{\frac{\alpha}{1-\alpha}} (1+g_L). \tag{3.17 revisited}$$

Using g_T and g_Q, equation (3.29), allows us to identify δ from:

$$E\left[\mathsf{XK}_t\right] = \frac{g_Q}{\beta^\star} + g_T + \delta \tag{3.29 rev.}$$

Additionally, $r^\star$ may be inferred using the Gordon growth formula:

$$E\left[\mathsf{PD}_t\right] = \frac{\beta^\star(1+g_T)}{1-\beta^\star(1+g_T)} \tag{3.37 rev.}$$

We then estimate[21] α and μ using equations (3.30) and (3.34).

$$E\left[\mathsf{CapShare}_t\right] = \frac{\mu+\alpha-1}{\mu}, \tag{3.30 rev.}$$

$$E\left[\mathsf{AvgRet}_t\right] = \frac{\mu+\alpha-1}{\alpha}(r^\star + \delta + \frac{g_Q}{\beta^\star}). \tag{3.34 rev.}$$

4.2.3 Risk and discount rate

Concludingly, we estimate the probability of a shock p using equation (3.32):

$$E\left[\mathsf{rf}_t\right] = \frac{E_t\left[e^{(1-\theta)\chi_{t+1}}\right]}{\beta^\star E_t\left[e^{-\theta\chi_{t+1}}\right]} - 1 \tag{3.32 rev.}$$

And finally we derive β using our definition of the composite parameter, eq. (3.23):

$$\beta = \beta^\star \left((1+g_{PC})^{-\sigma} \left(E_t\left[e^{(1-\theta)\chi_{t+1}} \right] \right)^{\frac{1-\sigma}{1-\theta}} \right)^{-1} \tag{3.23 rev.}$$

[20] Using our expression for g_T from eq. (3.17) we may write the expression:

$$\begin{aligned} E[\mathsf{TFPgrowth}_t] &= s_L g_T - s_L g_L - (1-s_L) g_Q \\ &= s_L \left(\frac{1}{1-\alpha} g_Z + \frac{\alpha}{1-\alpha} g_Q \right) - (1-s_L) g_Q \\ &= \frac{s_L}{1-\alpha} g_Z + \left(\frac{s_L \alpha}{1-\alpha} - (1-s_L) \right) g_Q, \end{aligned}$$

which is seen to simplify to g_Z when $s_L = 1-\alpha$.

[21] Using that $\beta^\star = \frac{1}{1+r^\star}$

4.3 Decomposition

We conduct a decomposition of the contribution from each parameter on the total change to a given moment from the first period to the second. The combined tool build on iPython calibrating the model and orchestrating the decomposition may be downloaded from `https://norgaardpetersen.dk/brink/`.

When changing a parameter from its initial to its final value, we face the issue of determining at what value to evaluate the other parameters, due to the non-linearity of the model.

Because of non-linearity, the change in a single parameter is dependent on its covariates. Had the model been linear, the covariation with other parameters would be zero and an additive decomposition would be unproblematic. Alternatively, we could linearise the model if the changes in parameters had been small. In general, however, we are left with no solution.

One might conduct a sequential decomposition changing one parameter at a time, which would indeed be additive (i.e. the sum of all contributions would equal to total change). This method, however, is path-dependent and in our case there is no natural order in which to change parameters. We wish to use a method that is path-*independent*, which is why we consider *all possible* orders in which to change parameters, as described below.

4.3.1 Theoretical framework

In this section we deliver a brief overview to help the intuition of our decomposition (and in particular why it is additive). For a technical description we refer to Farhi and Gourio (2018) and Gourio and Schulhofer-Wohl (2018)[22]. This example is borrowed from Druedahl (2019), using the notation of Biewen (2012).

Example with three parameters Let $f : [0,1]^3 \to \mathbb{R}$ and define

$$f_{\alpha\beta\gamma} \equiv f(\alpha, \beta, \gamma)$$

We wish to decompose the change $f_{111} - f_{000}$, which can be done in $3! = 6$ different ways:

$$\begin{aligned}
f_{111} - f_{000} &= (f_{111} - f_{011}) + (f_{011} - f_{001}) + (f_{001} - f_{000}) \\
f_{111} - f_{000} &= (f_{111} - f_{011}) + (f_{011} - f_{010}) + (f_{010} - f_{000}) \\
f_{111} - f_{000} &= (f_{111} - f_{101}) + (f_{101} - f_{001}) + (f_{001} - f_{000}) \\
f_{111} - f_{000} &= (f_{111} - f_{101}) + (f_{101} - f_{100}) + (f_{100} - f_{000}) \\
f_{111} - f_{000} &= (f_{111} - f_{110}) + (f_{110} - f_{100}) + (f_{100} - f_{000}) \\
f_{111} - f_{000} &= (f_{111} - f_{110}) + (f_{110} - f_{010}) + (f_{010} - f_{000})
\end{aligned}$$

Taking the sum of and rearranging yields:

$$\begin{aligned}
6(f_{111} - f_{000}) =& 2(f_{111} - f_{011}) + (f_{011} - f_{001}) + (f_{011} - f_{010}) + 2(f_{001} - f_{000}) \\
&+ 2(f_{111} - f_{101}) + (f_{101} - f_{001}) + (f_{101} - f_{100}) + 2(f_{010} - f_{000}) \\
&+ 2(f_{111} - f_{110}) + (f_{110} - f_{100}) + (f_{110} - f_{010}) + 2(f_{100} - f_{000}),
\end{aligned}$$

[22] We thank François Gourio and Sam Schulhofer-Wohl for sharing this mimeo.

implying that each contribution is weighted in the following way:

$$\begin{aligned}
f_{111} - f_{000} =& \frac{1}{3}(f_{111} - f_{011}) + \frac{1}{6}(f_{011} - f_{001}) + \frac{1}{6}(f_{011} - f_{010}) + \frac{1}{3}(f_{001} - f_{000}) \\
&+ \frac{1}{3}(f_{111} - f_{101}) + \frac{1}{6}(f_{101} - f_{001}) + \frac{1}{6}(f_{101} - f_{100}) + \frac{1}{3}(f_{010} - f_{000}) \\
&+ \frac{1}{3}(f_{111} - f_{110}) + \frac{1}{6}(f_{110} - f_{100}) + \frac{1}{6}(f_{110} - f_{010}) + \frac{1}{3}(f_{100} - f_{000}),
\end{aligned}$$

where we define:

$$\begin{aligned}
\Delta_1 =& \frac{1}{3}(f_{111} - f_{011}) + \frac{1}{6}(f_{011} - f_{001}) + \frac{1}{6}(f_{011} - f_{010}) + \frac{1}{3}(f_{001} - f_{000}) \\
\Delta_2 =& \frac{1}{3}(f_{111} - f_{101}) + \frac{1}{6}(f_{101} - f_{001}) + \frac{1}{6}(f_{101} - f_{100}) + \frac{1}{3}(f_{010} - f_{000}) \\
\Delta_3 =& \frac{1}{3}(f_{111} - f_{110}) + \frac{1}{6}(f_{110} - f_{100}) + \frac{1}{6}(f_{110} - f_{010}) + \frac{1}{3}(f_{100} - f_{000})
\end{aligned}$$

Using this, we can interpret Δ_i as the contribution from the i'th parameters on the total change $f_{111} - f_{000}$.

Weights in the general case Let $\theta = (\theta_1, \theta_2, ..., \theta_p)$. In the above, we set the number of parameters $p = 3$ and constrained $\theta_i \in \{0,1\}$, which may be thought of as the parameter taking the initial or final value. Weights as obtained above may be calculated as:

$$w(\theta_i) = \frac{j!(p-1-j)!}{p!},$$

where j is the number of parameters in θ which are at their initial value. Regarding only paths that begin at $f(0,0,...0)$ and end at $f(1,1,...,1)$ we find that there are $j!$ ways of reaching $f(1,1,..,\theta_i = 1,..,0,0) - f(1,1,..,\theta_i = 0,..,0,0)$ from $f(0,0,...,0)$ and then $(p-1-j)!$ ways to reach $f(1,1,...,1)$ from there.

A note on decompositioning The above described method neatly ascribes a certain fraction of the total change in the moment to each individual parameter and additionally ensure that the sum of the individual contributions (satisfyingly) equals the total change. However, the reader should be alert, that despite circumventing certain issues to non-linearity of the model, the method of decomposition does not *solve* them. As Biewen (2012) notes there's no sound argument as to why the individual contributions should sum to the total change. In contrast, one may argue that due to the interaction between factors it should not hold, and we ask the reader to allocate a considerable amount of attention to this aspect when interpreting the results.

However, the above method of decomposition *is* path-independent which means that it does not depend on the order in which parameters are changed from initial to final values, compared to say a sequential decomposition, and further it is comprehensive, meaning it entails all counterfactuals.

5 Empirical results for Great Britain

We present all our empirical results from the cross country analysis in the appendix. As for this section, we once again turn our focus to Great Britain, the largest European economy with consistent time series data available. We first report and comment parameter estimates characterizing the economy for each of the two subsamples, then turn to a decomposition of the observed changes in the data moments in order to highlight the importance of each parameter change. Finally, a few comments on the cross country evidence are presented.

Table 3: Estimated parameters for two subsamples, Great Britain

		Estimates		
Parameter name	Symbol	1984 - 1999	2000 - 2015	Change
Discount factor	β	0.953	0.965	0.012
Markup	μ	1.181	1.203	0.023
Disaster probability	p	0.013	0.042	0.029
Depreciation, pct.	δ	4.177	4.277	0.100
Cobb-Douglas parameter	α	0.352	0.303	-0.049
Population growth, pct.	g_L	0.255	0.652	0.396
TFP growth, pct.	g_Z	1.560	0.798	-0.762
Technological change, pct.	g_Q	-0.231	0.042	0.272
Labour Supply	$\bar{N}$	0.448	0.472	0.024

Notes: Change is the absolute difference between the first and second subsample.

5.1 Comparing the two subsamples

We present the estimated parameter values for each subsample, along with the change between periods, in Table 3. The results give weight to a range of common explanations – we turn to these shortly – describing the decline in the real risk-free rate.

As we aim at explaining shifts in the economy, the *change between periods* is the estimate of most interest. From the first to second subsample, the discount factor, β, has increased by 1.2 percentage points, indicating that agents in the second subsample place higher value on future consumption compared to the agents of the first subsample. This leads to higher savings, and thus a larger savings supply – a common explanation to the decreasing real risk-free interest rates, emphasized by our results.

The markup, and thus market power of firms, in the economy has grown by 2.3 percentage points. As commonly done, we attribute this to a declining (or less effective) competition in the economy, leading to higher profits for companies and, in turn, increasing average return to capital. However, interestingly, the Cobb-Douglas parameter, α, decreases quite dramatically by five points. This leads to an increase in the income share to labour compensation of three percentage points[23]. Correspondingly, the income share to capital decreases by

[23]As noted previously, an economy with mark-ups leads to shares of income given by $s_L = \frac{1-\alpha}{\mu}$, $s_C = \frac{\alpha}{\mu}$ and $s_\Pi = \frac{\mu-1}{\mu}$. Using these, we find $s_L = 0.55$, $s_C = 0.30$ and $s_\Pi = 0.15$ in the first sub-period and $s_L = 0.58$, $s_C = 0.25$ and $s_\Pi = 0.17$ in the second.

three percentage points. This covers a decrease in capital share, S_C, of five percentage points and an increase in profits to rents of two points. In addition, as α is output elasticity to capital, we find that relative productivity of capital decreases between periods.

The reason for these findings is in the fitting of the model. To account for the observed decline in labour share, a large decrease in α is needed, given $s_L = \frac{1-\alpha}{\mu}$. This particular trend could also be explained by a decrease in the markups, μ, but this would conflict with the stable average return to capital observed.

The macroeconomic risk of a large[24] shock to the economy has more than tripled from 1.3% to 4.2%, accelerating the risk premium. This is somewhat contrary to the notion of a Great Moderation in advanced economies over the past decades with declining volatility in i.a. output, inflation rates and unemployment. On the other hand, the crisis of 2008 as well as the dot-com bubble in 2001 may explain why we find a larger risk in the second subsample than in the first.

The increase in risk is perhaps the most common explanation put forward when accounting for the increasing wedge between the return to capital and the real risk-free interest rate, and our analysis adds weight to this argument but, importantly, does not ascribe the entire gap to risk, as especially markups also play a role.

Depreciation of physical capital has remained remarkably stable at just above 4%. The remaining parameters are chosen directly as their counterpart in the dataset, and we find that labour supply and population growth has increased, while total factor productivity in the second period is half of that in the first. For a more thorough description of these parameters, we refer to the data description, Section 2.

The above section described a range of characteristics in the economy for each period, but did not comment on the *importance* of each parameter when accounting for the trends observed in data; something we turn to next.

5.2 Decomposing the changes

In order to highlight the effects of each parameter, we perform a decomposition as described in Section 4.3. The results —that is, the effect on each of the data moments, given the estimated change in the parameter— is presented in Table 4. The nine parameter effects for a single moment sums to the observed change in data[25]. As expected, we find that the increase in savings supply between periods (caused by the increase in β as reported in Table 3) leads to a decrease in real risk-free interest rate. However, we find that the same increase in savings supply concurrently decreases average return to capital *more* than the interest rate, and thus that the spread between gross profitability and real risk-free interest rate decreases as a result of the larger savings supply – a quite important result.

The largest individual effect on the spread comes from the increase in macroeconomic risk. Risk lowers the risk-free interest rate and increases average return to capital, thus widening the gap from both the upper and lower bound. As

[24]As mentioned in 4, we use a zero-mean shock of 15% in either direction

[25]This is not a coincidence, rather it is a result of carefully chosen weights in the decomposition, as described in Section 4.3

evident, from the last row of Table 4, we find that the estimated increase in risk accounts for more than two thirds of the increase in the spread, ceteris paribus. The increased markups adds to the spread, as expected c.f. section 3.5. As does the estimated decrease in α and g_Q, while the fall in total factor productivity growth decreases the gap.

Note that the effects on the price to dividend ratio are quite high from each of the parameters[26]. This explains why the entire decline in real risk-free interest rate cannot be ascribed savings supply – it would make it impossible to match the observed PD-ratio. In particular, the increase in β alone implies an increase by around 50%, indicating that stock prices should soar between periods. This is not the case – the result is counterfactual as it ignores the fall in growth of the economy, and thus the decreasing growth of dividends. As evident, the falling growth in total factor productivity offsets the increase in the price to dividend ratio. Further, higher risk works through the increase in interest rate, $r^\star$, which lowers prices of shares due to higher required return. This underlines the importance and relevance of modeling an entire macroeconomic framework when describing general macro-finance trends – as prevalent, more naïve approaches may lead to mistaken results.

5.3 Cross country evidence

In addition to the results for Great Britain, we have conducted the empirical analysis for a range of countries, including Denmark, a selection of EU member states, USA, and Japan. The results for Denmark are shown and commented in section H of the appendix, while the estimated parameters and the decomposition are shown for the rest of the countries in section I of the appendix. Note critically that our framework models a closed economy and thus the insights, particularly for small, open economies, are prone to be erroneous, why they should be interpreted cautiously.

In general, the results found for Great Britain are evident across countries. The gap between average return to capital and risk-free interest rate increases in all countries. In addition, all economies experience increases in savings supply and macroeconomic risk, while mark-up increases in all economies except for Denmark and Italy.

As a result of the increasing labour share in Great Britain – a trend unique to this economy – a single result does not hold across countris: The estimated decrease in the Cobb-Douglas parameter, α. The Cobb-Douglas parameter increases or remains stable for all economies but the British.

For the small, open economies – especially Denmark, Finland and the Netherlands – we find high increases in depreciation rate. This is likely a result of modelling these economies as closed: The interest rate faced by these economies is however exogenously given and does not respond to domestic changes in preferences, leading to an inexplicable high capital ratio, at least if these countries save more than the countries around them. This is indeed the case – on average, all three economies run surpluses on their current accounts over the sample period, and to a larger degree in the second subsample. Enforcing the framework onto these economies thus requires an (unnaturally) high increase in depreciation rate to account for the actual capital stock.

[26]This is also, in part, a scaling effect – the PD-ratio is magnitudes higher than any of the other parameters.

Table 4: Decomposition of changes in data

	Data			*Decomposition of* Δ								
	P1	P2	Δ	β	μ	p	δ	α	g_L	g_Z	g_Q	$\bar{N}$
Average Return to Capital	15.63	15.63	−0.00	−2.08	0.68	0.84	0.16	0.78	0.00	−0.94	0.57	0.00
Gross Capital Share	45.15	42.06	−3.08	0.00	1.07	0.00	0.00	−4.15	0.00	0.00	0.00	0.00
Risk Free Interest Rate	5.11	1.33	−3.77	−1.28	0.00	−1.92	0.00	−0.06	0.00	−0.58	0.07	0.00
Price-Dividend Ratio	26.68	31.78	5.10	11.65	0.00	−4.92	0.00	−0.59	3.81	−5.50	0.65	0.00
Investment-Capital Ratio	6.49	6.14	−0.35	0.00	0.00	0.00	0.10	−0.12	0.40	−1.14	0.41	0.00
TFP Growth	1.36	0.66	−0.70	0.00	−0.02	0.00	0.00	−0.00	0.00	−0.65	−0.04	0.00
Investment Price Growth	0.23	−0.04	−0.27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	−0.27	0.00
Population Growth	0.26	0.65	0.40	0.00	0.00	0.00	0.00	0.00	0.40	0.00	0.00	0.00
Employment-Population Ratio	44.80	47.25	2.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.44
Spread	10.52	14.30	3.77	−0.80	0.68	2.76	0.16	0.84	0.00	−0.36	0.50	0.00

6 Discussion

Our theoretical framewor has a number of limitations. Importantly, we apply a closed-economy framework to open economies. This approximation is especially troublesome for small, open economies such as Belgium, Denmark and the Netherlands, with exports to GDP ratios of 57%, 85% and 86%[27], respectively.

In addition hereto, this section discusses the approach of estimating return to capital and equity from a finance-based measure, the interpretation of the savings supply in the economy, and how common trends affect economies. We lastly address a few additional concerns.

6.1 Imputing the return on capital

Using financial measures to impute rate of return In our model we identify the required rate of return $r^\star$ using the Gordon growth formula as first presented in Gordon and Shapiro (1956). Importantly, Gordon and Shapiro argue that identification of the required return on equity r^E requires knowledge on the expected future dividends[28]. If we assume dividends grow at constant rate g we may write the relation between stock prices P and dividends D as:

$$P_0 = \sum_{t=1}^{\infty} \frac{D_0(1+g)^t}{(1+r^E)^t} = \frac{D_0(1+g)}{r^E - g}.$$

In our application, we assume that growth in dividends equal growth in output, $g = g_T$, and that required return on equity equals that on capital, leading to

$$\frac{P}{D} = \frac{1+g_T}{r^\star - g_T}, \qquad \text{or} \qquad r^\star = \frac{D(1+g_T)}{P} + g_T.$$

In our calibration, we impute the required return on capital using 15-year averages of share prices and dividends under the assumption that growth in dividends equal that of output growth. Natural questions are: How well does 15-year average values of output growth approximate expected future dividend growth? And is it fair to equate required return on equity with that on capital?

Neglecting variability A possible critique of this approach is that it neglects large variations in the price dividend ratio as evident from the sixth panel of Figure 2,

For instance, we find that whether we place year 2000, and hence the peak of the dot-com bubble, in the first or second subsample affects our estimation of perceived probability of a shock fairly much. In our baseline estimation, we find an increase in risk p between periods of 2.9 percentage points. Changing the subsamples such that the first subsample ends in 2001, rather than 1999, we find an increase in risk of 4.1 percentage points between periods – thus quite a difference depending on where we place the peak of the dot-com bubble.

[27] European Commission's Directorate General for Economic and Financial Affairs 2018, tables: OVGD, OXGS.

[28] For convenience of notation we define the net return on equity as $r^E = R^E - 1$ using terms from Section 3.3.6.

In addition hereto, one might pose the sensible hypothesis that in the years leading up the dot-com bubble many companies with high prices and low dividend payments flowed into the stock markets, followed by the reverse phenomena after the dot-com bubble. These points, and the previously mentioned risk of survival bias in the index indicates that our estimation of $r^\star$ may be flawed.

Equating return on equity and capital As briefly touched upon in the model, Section 3.3.6, we utilize the neoclassical standard assumption that the no-arbitrage condition holds, in order to justify matching the rate of return on capital and equity. This is not an undemanding assumption. As e.g. Gomme, Ravikumar, and Rupert (2011) show, discrepancies between return on equity and return on capital exist, both when looking at average returns, but especially with regards to volatility.

Further, as we use a finance-based measure, the price dividend ratio, to identify the interest rate of a *closed* economy, we implicitly assume that the financial market is closed to the rest of the world. This is hardly the case for Great Britain; according to Crane (2014), foreign investors held 54% of British stocks at the end of our sample period.

6.2 Changes in savings supply

We have previously interpreted an increase in the parameter β as an increase in the savings supply. As β denotes the 'impatience' of agents (see eq. (3.1)), an increase in β means that agents value future consumption higher and hence save more. Note, however, that other factors than preference for future consumption may affect savings supply. As we do not model savings supply explicitly in the model we must be aware that other effects may affect the savings supply and eventually turn up in the estimate of β. We briefly discuss these here.

Influx of capital or domestic trends? As our framework models a closed economy, it does not allow for the increase in savings supply to be the result of an influx of foreign capital. Yet, Caballero, Farhi, and Gourinchas (2017) argue that the observed trends of stable return to capital and decreasing interest rates may be the result of a demand for safe assets exceeding the supply of such. They point to the fact that high-saving emerging economies demand safe assets, but that such assets are mostly found in advanced economies with lower growth rates than the emerging economies implies a structural excess demand leading to further decreasing returns on safe assets. This effect, if true, would however be disguised in our estimate of β as a change in domestic savings.

The case for demographics A different argument can be made that domestic trends, such as demographic changes, increases the savings supply. Following Carvalho, Ferrero, and Nechio (2016), increased life-expectancy could result in higher savings for retirement, thus also increasing the savings supply. This would also be disguised in the estimate of β.

In sum, compelling cases can be made that the savings supply is affected by other factors than impatience of agents in the model, and while this may also play a part in the empirical results we obtain, we must be cautious to attribute the increase of savings supply solely to impatience of agents.

6.3 Common trends

As evident from the time series presented in this paper and Section H and I of the appendix, many of the macroeconomic trends are shared across developed economies. This implies that results from individual countries may, to some degree, be used as a proxy for the global economy. This is an appealing theory, as we may disregard the controversies from applying a closed-economy model to open economies – the global economy being inherently closed.

With the data at hand, an interesting experiment to consider is to create an aggregate compound of all countries. However, taking simple averages across countries and redoing the empirical analysis does not yield fruitful results – neither should it necessarily, considering that the derived relations may not hold. Keeping the work of Caballero, Farhi, and Gourinchas (2017) in mind, proxying the global economy with local data should not be done without consulting data on high-saving, emerging economies.

6.4 Other concerns

Is the assumption of equal shocks sane? Our model includes the rather implausible assumption, that any shock to the productivity of the economy is fully countered by a shock to capital accumulation. This is not an assumption based on evidence or intuition, nor does it claim to be. Rather, it is an assumption that allows us to solve the model by keeping the economy on the risky balanced growth path. It is for the same reason that we do not consider transitional dynamics in the economy. We argue that little is lost by this simplification, as we measure the economy in the (medium-) long run.

Data concerns For Great Britain, we have constructed a composite dataset (excluding Northern Ireland), combining series from various sources, including the OECD, Penn World Table and DataStream. When redoing the empirical analysis, we find no remarkable differences in parameter estimates or the result of the decomposition. This may be due to our approach – taking averages over time will likely be robust to minor differences in the underlying data.

Robustness checks In all presented applications, we assume the risk aversion parameter $\theta = 12$, and the intertemporal elasticity of substitution to be 2 (i.e. $\sigma = 0.5$). Table 5 reports the parameter estimates for our baseline model, for baseline with $\theta = 18$, and for baseline with $\sigma = 2$.

We note that IES does not affect our results remarkably – only the savings supply slightly. Changing the level of risk aversion only affects probability of a risk, which decreases. However, the same change in risk between periods persist.

Changing preferences Finally, we may ask: Are we able to model the results of Caballero, Farhi, and Gourinchas (2017) as described above within our framework? One way of interpreting the increasing demand for safe assets might be as a change in risk aversion from one period to the other when modelling the global economy. This would, however, violate our implicit assumption of stable preferences with severe effects for our model as any change in behaviour may be justified through a change in preferences or context. For further on this point, we refer to Reiss (2013, ch.3) and Becker (1978).

Table 5: Robustness checks

		Baseline		$\theta = 18, \sigma = 0.5$		$\theta = 12, \sigma = 2$	
Parameter name	Symbol	1984 - 1999	2000 - 2015	1984 - 1999	2000 - 2015	1984 - 1999	2000 - 2015
Discount factor	β	0.953	0.965	0.952	0.963	0.979	0.960
Mark-up	μ	1.181	1.203	1.181	1.203	1.181	1.203
Disaster probability	p	0.013	0.042	0.005	0.016	0.013	0.042
Depreciation, pct.	δ	4.177	4.277	4.177	4.277	4.177	4.277
Cobb-Douglas parameter	α	0.352	0.303	0.352	0.303	0.352	0.303
Population growth, pct.	g_L	0.255	0.652	0.255	0.652	0.255	0.652
TFP growth, pct.	g_Z	1.560	0.798	1.560	0.798	1.560	0.798
Technological change, pct.	g_Q	-0.231	0.042	-0.231	0.042	-0.231	0.042
Labour Supply	$\bar{N}$	0.448	0.472	0.448	0.472	0.448	0.472

7 Conclusion

This paper investigates the increasing gap between interest rates and the average return to capital, observed over the past decades in most advanced economies. We base our analysis on Great Britain, but extend the empirical analysis to cover a wider range of countries. By using an augmented neoclassical growth model that incorporates risk, imperfect competition and allows for investment specific technological change, we find that a combination of factors jointly explain the widening gap – in particular higher markups and increased risk. Technological change has little effect. An increase in savings supply pushes the interest rate downwards, but also the average return to capital, decreasing the gap. We generally obtain similar results for other countries examined, though details vary slightly dependent on the characteristics of each country.

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Appendices

A Construction of data series

Source All time series except one are sourced from AMECO[29], containing data for the EU as well as certain other OECD countries. The spring 2019 update implied large revisions to certain dataseries in our dataset for which we have yet to receive documentation. For this reason we apply data from the 2018 Fall edition of AMECO.

Data for dividend yield, used to calculate the price dividend ratio, is sourced from[30] `macrohistory.net`.

Nomenclature We note time series names using typewriter font. We note that AMECO apply different names for every country using a three letter prefix, consiting of the country's ISO-abbreviation. Hence, a time series for Great Britain reads e.g. `GBR.3.1.0.0.PIGT`. Each section below describes the computation of individual time series, using shorthands as they appear in the dataset available in our online repository.

PopGrowth Population growth is calculated as the year-on-year percentage growth in total population, series {`ISO`}`.1.0.0.0.NPTD`.

PriceInvt The growth in investment prices are computed building on Gomme, Ravikumar, and Rupert (2011, p. 267)[31] by dividing the price deflator for gross capital formation {`ISO`}`.3.1.0.0.PIGT` by the consumer price index {`ISO`}`.3.0.0.0.ZCPIN` and taking year-on-year percentage growth.

EmpPop The employment-population ratio is calculated as employment measured in persons {`ISO`}`.1.0.0.0.NETD` divided by total population {`ISO`}`.1.0.0.0.NPTD`. Due to lack of data on Great Britain we may not take into account the share of the population outside the workforce (e.g. persons below the age of 16 years). This doesn't affect our results for other countries where data has been available. Thus, for consistency in measures, we maintain a common definition throughout.

TFPgrowth Calculated as year-on-year percentage change in total factor productivity, series {`ISO`}`.3.0.0.0.ZVGDF`

CapShare Gross capital share is calculated as one minus the labour share, ie.

$$\frac{\Pi}{Y} = 1 - s_L$$

using adjusted wage share as a percentage of GDP at current market prices, series {`ISO`}`.1.3.0.0.ALCD0`

[29] European Commission's Directorate General for Economic and Financial Affairs. 2018. *Annual macro-economic database of the european commission (ameco).* Available for download at `https://ec.europa.eu/info/sites/info/files/economy-finance/ameco_autumn20181.zip`

[30] Òscar Jordà et al. 2019. The rate of return on everything, 1870–2015. *The Quarterly Journal of Economics* 134 (3): 1225–1298

[31] We are aware that Farhi and Gourio (2019) construct the investment price growth as: "the growth of the ratio of the chained index for fixed investment (jf@usna) to the chained index of nondurable consumption and services."

XK The investment-capital ratio is calculated gross fixed capital formation at constant prices {ISO}.1.1.0.0.OIGT divided by the net capital stock at constant prices, series {ISO}.1.0.0.0.OKND

PD The price dividend ratio is calculated as the inverse of the Equity Dividend Yield, series eq_dp. Data is sourced from Jordà et al. (2019).

AvgRet Gross profitability is computed as CapShare divided by the capital-output ratio:

$$\frac{s_K Y}{K} = \frac{s_K}{K/Y}.$$

We apply the net capital stock per unit of GDP, series {ISO}.1.3.0.0.AKNDV

rf The risk free rate is given as the short term real interest rate, deflated by the consumption deflator, series {ISO}.1.3.0.0.ISRC. In most cases, the short term risk-free interest rate is measured by the 3-month interbank rate.

B Derivation of Euler Equation

The utility function is given in the following way, where we may consider V a function of K_t, as the latter is the system's state variable:

$$V(K_t) = \left((1-\beta)C_t^{1-\sigma} + \beta E_t[V(K_{t+1})^{1-\theta}]^{\frac{1-\sigma}{1-\theta}}\right)^{\frac{1}{1-\sigma}}. \tag{B.1}$$

The resource constraint reads:

$$Y_t = X_t + C_t \qquad \Longleftrightarrow \qquad X_t = Y_t - C_t$$

Further, we may write total income as[32]:

$$Y_t = R_t K_t + w_t L_t + \Pi$$

Substituting the above equations into capital accumulation yields:

$$K_{t+1} = ((1-\delta)K_t + Q_t(R_t K_t + w_t N_t + \Pi - C_t)e^{\chi_{t+1}}, \tag{B.2}$$

First order condition As the utility function includes utility in all future periods, households maximise utility by choosing an optimal level of consumption in the current period (and thus implicitly choosing the future level of capital). This gives rise to the first order condition:

$$\begin{aligned}
0 &= \frac{\partial V(K_t)}{\partial C_t} \\
&= \frac{1}{1-\sigma} V(K_t)^{\frac{1}{1-\sigma}-\frac{1-\sigma}{1-\sigma}} \Bigg[(1-\beta)(1-\sigma)C_t^{-\sigma} + \beta\frac{1-\sigma}{1-\theta} E_t[V(K_{t+1})^{1-\theta}]^{\frac{\theta-\sigma}{1-\theta}} \\
&\qquad \cdot (1-\theta)E_t\left[V(K_{t+1})^{-\theta} \cdot \frac{\partial V(K_{t+1})}{\partial C_t}\right]\Bigg] \\
&= V(K_t)^{\sigma}\Bigg[(1-\beta)C_t^{-\sigma} + \beta E_t[V(K_{t+1})^{1-\theta}]^{\frac{\theta-\sigma}{1-\theta}} \\
&\qquad \cdot E_t\left[V(K_{t+1})^{-\theta} \cdot \frac{\partial V(K_{t+1})}{\partial C_t}\right]\Bigg] \\
&= V(K_t)^{\sigma}\Bigg[(1-\beta)C_t^{-\sigma} + \beta E_t[V(K_{t+1})^{1-\theta}]^{\frac{\theta-\sigma}{1-\theta}} \\
&\qquad \cdot E_t\left[V(K_{t+1})^{-\theta} \cdot \frac{\partial V(K_{t+1})}{\partial K_{t+1}}\frac{\partial K_{t+1}}{\partial X_t}\frac{\partial X_t}{\partial C_t}\right]\Bigg] \\
&= V(K_t)^{\sigma}\Bigg[(1-\beta)C_t^{-\sigma} - \beta E_t[V(K_{t+1})^{1-\theta}]^{\frac{\theta-\sigma}{1-\theta}} \\
&\qquad \cdot E_t\left[V(K_{t+1})^{-\theta} \cdot \frac{\partial V(K_{t+1})}{\partial K_{t+1}} Q_t e^{\chi_{t+1}}\right]\Bigg]
\end{aligned}$$

[32] Where compensation to labour $w_t N_t = s_L Y_t = \frac{1-\alpha}{\mu} Y_t$, total returns to capital $R_t K_t = s_C Y_t = \frac{\alpha}{\mu} Y_t$ and pure profits $\Pi = s_\Pi Y_t = \frac{\mu-1}{\mu} Y_t$

We rearrange to obtain

$$\begin{aligned}(1-\beta)C_t^{-\sigma} &= \beta E_t[V(K_{t+1})^{1-\theta}]^{\frac{\theta-\sigma}{1-\theta}} \cdot E_t\left[V(K_{t+1})^{-\theta} \cdot \frac{\partial V(K_{t+1})}{\partial K_{t+1}} Q_t e^{\chi_{t+1}}\right] \\ &= \beta E_t\left[E_t[V(K_{t+1})^{1-\theta}]^{\frac{\theta-\sigma}{1-\theta}} \cdot V(K_{t+1})^{-\theta} \cdot \frac{\partial V(K_{t+1})}{\partial K_{t+1}} Q_t e^{\chi_{t+1}}\right]\end{aligned} \tag{B.3}$$

Envelope condition The above expression includes the effect on utility caused by a change in capital level (both in period $t+1$), but note that this is subject to (constant) optimization by households, which is why we use the envelope theorem to incur

$$\begin{aligned}\frac{\partial V(K_t)}{\partial K_t} &= \frac{1}{1-\sigma} V(K_t)^{\frac{1}{1-\sigma}-\frac{1-\sigma}{1-\sigma}} \cdot \beta\frac{1-\sigma}{1-\theta} E_t\left[V(K_{t+1})^{1-\theta}\right]^{\frac{\theta-\sigma}{1-\theta}} \\ &\qquad \cdot (1-\theta) E_t\left[V(K_{t+1})^{-\theta} \cdot \frac{\partial V(K_{t+1})}{\partial K_t}\frac{\partial V(K_{t+1})}{\partial K_{t+1}}\right] \\ &= V(K_t)^{\sigma} \cdot \beta E_t\left[V(K_{t+1})^{1-\theta}\right]^{\frac{\theta-\sigma}{1-\theta}} \\ &\qquad \cdot E_t\left[V(K_{t+1})^{-\theta} \cdot \frac{\partial V(K_{t+1})}{\partial K_t}\frac{\partial V(K_{t+1})}{\partial K_{t+1}}\right].\end{aligned}$$

Going forward, we move the first expectation value within the second as it may be regarded as constant

$$\begin{aligned}&= V(K_t)^{\sigma} \cdot \beta E_t\left[E_t\left[V(K_{t+1})^{1-\theta}\right]^{\frac{\theta-\sigma}{1-\theta}}\right. \\ &\qquad \left. \cdot V(K_{t+1})^{-\theta} \cdot \frac{\partial V(K_{t+1})}{\partial K_t}\frac{\partial V(K_{t+1})}{\partial K_{t+1}}\right] \\ &= V(K_t)^{\sigma} \cdot \beta E_t\left[E_t\left[V(K_{t+1})^{1-\theta}\right]^{\frac{\theta-\sigma}{1-\theta}}\right. \\ &\qquad \left. \cdot V(K_{t+1})^{-\theta} \cdot ((1-\delta) + R_t Q_t)\, e^{\chi_{t+1}} \frac{\partial V(K_{t+1})}{\partial K_{t+1}}\right] \\ &= V(K_t)^{\sigma} \cdot \beta E_t\left[E_t\left[V(K_{t+1})^{1-\theta}\right]^{\frac{\theta-\sigma}{1-\theta}}\right. \\ &\qquad \left. \cdot V(K_{t+1})^{-\theta} \cdot \left(\frac{1-\delta}{Q_t} + R_t\right) Q_t e^{\chi_{t+1}} \frac{\partial V(K_{t+1})}{\partial K_{t+1}}\right].\end{aligned}$$

Variables with subscript t are known within the period allowing us to move them outside expectation value, ultimately yielding

$$\begin{aligned}\frac{\partial V(K_t)}{\partial K_t} &= V(K_t)^{\sigma} \cdot \beta E_t\left[E_t\left[V(K_{t+1})^{1-\theta}\right]^{\frac{\theta-\sigma}{1-\theta}}\right. \\ &\qquad \left. \cdot V(K_{t+1})^{-\theta} \cdot \frac{\partial V(K_{t+1})}{\partial K_{t+1}} Q_t e^{\chi_{t+1}}\right]\left(\frac{1-\delta}{Q_t} + R_t\right).\end{aligned}$$

Combining Using the above and the expression for $(1-\beta)C_t^{-\sigma}$ from equation (B.3), we write

$$\frac{\partial V(K_t)}{\partial K_t} = V(K_t)^{\sigma}(1-\beta)C_t^{-\sigma}\left(\frac{1-\delta}{Q_t} + R_t\right),$$

which we lead one period to obtain

$$\frac{\partial V(K_{t+1})}{\partial K_{t+1}} = V(K_{t+1})^{\sigma}(1-\beta)C_{t+1}^{-\sigma}\left(\frac{1-\delta}{Q_{t+1}} + R_{t+1}\right).$$

Reinserting this leaded expression into the first order expression yields

$$\begin{aligned}
&(1-\beta)C_t^{-\sigma} \\
&\quad= \beta E_t\Big[E_t[V(K_{t+1})^{1-\theta}]^{\frac{\theta-\sigma}{1-\theta}} \cdot V(K_{t+1})^{-\theta} \\
&\qquad\qquad \cdot V(K_{t+1})^{\sigma}(1-\beta)C_{t+1}^{-\sigma}\left(\frac{1-\delta}{Q_{t+1}} + R_{t+1}\right)Q_t e^{\chi_{t+1}} \\
&\quad= \beta E_t\left[\left(\frac{1}{E_t[V(K_{t+1})^{1-\theta}]^{\frac{1}{1-\theta}}}\right)^{\sigma-\theta}\right. \\
&\qquad\qquad \cdot V(K_{t+1})^{\sigma-\theta}(1-\beta)C_{t+1}^{-\sigma}\left(\frac{1-\delta}{Q_{t+1}} + R_{t+1}\right)Q_t e^{\chi_{t+1}} \\
&\quad= \beta E_t\left[\left(\frac{V(K_{t+1})}{E_t[V(K_{t+1})^{1-\theta}]^{\frac{1}{1-\theta}}}\right)^{\sigma-\theta}\right. \\
&\qquad\qquad \cdot (1-\beta)C_{t+1}^{-\sigma}\left(\frac{1-\delta}{Q_{t+1}} + R_{t+1}\right)Q_t e^{\chi_{t+1}},
\end{aligned}$$

which we rearrange to obtain

$$\begin{aligned}
1 = E_t&\left[\beta\left(\frac{V(K_{t+1})}{E_t[V(K_{t+1})^{1-\theta}]^{\frac{1}{1-\theta}}}\right)^{\sigma-\theta}\right. \\
&\qquad \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{1-\delta}{Q_{t+1}} + R_{t+1}\right)Q_t e^{\chi_{t+1}}
\end{aligned}.$$

We define

$$M_{t+1} \equiv \beta\left(\frac{V(K_{t+1})}{E_t[V(K_{t+1})^{1-\theta}]^{\frac{1}{1-\theta}}}\right)^{\sigma-\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \tag{B.4}$$

and note that

$$R_{t+1}^K = \left(\frac{1-\delta}{Q_{t+1}} + \frac{\alpha}{\mu}\frac{Y_t}{K_t}\right)Q_t e^{\chi_{t+1}}, \tag{B.5}$$

where

$$R_{t+1} = \frac{\alpha}{\mu}\frac{Y_t}{K_t}.$$

Consequently, this yields the Euler Equation

$$1 = E_t\left[M_{t+1}R_{t+1}^K\right]. \tag{EE}$$

C Epstein-Zin preferences

We apply Epstein-Zin preferences in our framework to enable the analysis of risk. We incorporate three parameters: The discount factor β, the inverse of the intertemporal elasticity of substituion (IES) σ and risk aversion θ.

Discount Factor The discount factor β determines agents' time preferences. The smaller β is, the less agents' value future consumption relative to current consumption.

Intertemporal elasticity of substitution IES is a measure of how willingly agents are to shifting consumption between periods. For low values of σ, ie. *elastic* intertemporal substitution, the effect on utility from changes in consumption is – ceteris paribus – less than the effect on utility (which is relatively more concave) for large values of σ, ie. *inelastic* intertemporal substitution (Romer 2012, p. 51). For instance an increase in the interest rate may result in a decrease in consumption as a result of increased return to savings (substitution effect) and an increase in consumption as a result of increased income (income effect).

Risk aversion parameter Risk aversion θ describes agents' attitude towards risk. Ceteris paribus, large values for θ imply greater risk aversion.

Motivation We may motivate using Epstein-Zin preferences as we need not assume that risk aversion is linked to the intertemporal elasticity of substitution as is the case for e.g. CRRA preferences. Further, for $\theta > \sigma$ (as we assume), meaning that risk aversion is larger than the inverse of IES, early resolution (of e.g. gambles) is preferred to late resolution, which is intuitively preferable. If agents' preferences are formulated as expected utility, agents are indifferent (Epstein and Zin 1989).

Special case In the special case where $\sigma = \theta$, Epstein-Zin preferences are equivalent to CRRA preferences, see Gilchrist (2013). Further, we note that for $\sigma = 1$ we recover log-preferences, meaning that the substitution effect and the income effect on consumption are of equal size.

Our assumptions Risk aversion and intertemporal elasticity of substitution is intertwined. For more details we refer to Epstein and Zin (1989, p. 952). In all our applications we assume that $\theta > 1$, in specific $\theta = 12$. In addition we assume $1 > \sigma > 0$ (specifically $\sigma = 0.5$, implicitly assuming that $\theta > \sigma$, i.e. preference for early resolution. Finally, it holds that $0 < \beta < 1$.

D Stochastic Discount Factor in equilibrium

Stochastic Discount Factor Firstly, we note that:

$$c_{pc,t} = \frac{C_t}{L_t} = \frac{c_t S_t T_t}{L_t}, \text{ and } V_{pc,t} = \frac{V_t}{L_t} = \frac{v_t S_t T_t}{L_t}$$

We have previously defined:

$$\begin{aligned}
M_{t+1} &\equiv \beta \left(\frac{c_{pc,t+1}}{c_{pc,t}} \right)^{-\sigma} \left(\frac{V_{pc,t+1}}{E_t[V_{pc,t+1}^{1-\theta}]^{\frac{1}{1-\theta}}} \right)^{\sigma-\theta} \\
&= \beta \left(\frac{\frac{S_{t+1}T_{t+1}c_{t+1}}{L_{t+1}}}{\frac{S_t T_t c_t}{L_t}} \right)^{-\sigma} \left(\frac{\frac{S_{t+1}T_{t+1}v_{t+1}}{L_{t+1}}}{E_t\left[\left(\frac{S_{t+1}T_{t+1}v_{t+1}}{L_{t+1}} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}} \right)^{\sigma-\theta}
\end{aligned}$$

We note that T, L and v are assumed deterministic, hence we treat them as constants, hence

$$= \beta \left(\frac{S_{t+1}}{S_t} \frac{c_{t+1}}{c_t} \frac{T_{t+1}}{T_t} \frac{L_t}{L_{t+1}} \right)^{-\sigma} \left(\frac{S_{t+1} \frac{T_{t+1}v_{t+1}}{L_{t+1}}}{\frac{T_{t+1}v_{t+1}}{L_{t+1}} E_t\left[(S_{t+1})^{1-\theta} \right]^{\frac{1}{1-\theta}}} \right)^{\sigma-\theta}$$

In equilibrium $c_{t+1} = c_t$ yielding

$$\begin{aligned}
&= \beta \left(e^{\chi_{t+1}} \frac{1+g_T}{1+g_L} \right)^{-\sigma} \left(\frac{S_{t+1}}{E_t\left[(S_{t+1})^{1-\theta} \right]^{\frac{1}{1-\theta}}} \right)^{\sigma-\theta} \\
&= \beta \left(\frac{1+g_T}{1+g_L} \right)^{-\sigma} \left(e^{\chi_{t+1}} \right)^{-\sigma} \left(\frac{(S_t e^{\chi_{t+1}})^{1-\theta}}{E_t\left[(S_{t+1})^{1-\theta} \right]} \right)^{\frac{\sigma-\theta}{1-\theta}} \\
&= \beta \left(\frac{1+g_T}{1+g_L} \right)^{-\sigma} \left(e^{\chi_{t+1}} \right)^{-\sigma} \left(e^{\chi_{t+1}} \right)^{\sigma-\theta} \left(\frac{E_t\left[(S_{t+1})^{1-\theta} \right]}{S_t^{1-\theta}} \right)^{\frac{\theta-\sigma}{1-\theta}}
\end{aligned}$$

In period t we may regard S_t as deterministic

$$= \beta \left(\frac{1+g_T}{1+g_L} \right)^{-\sigma} \left(e^{\chi_{t+1}} \right)^{-\theta} \left(E_t\left[e^{(1-\theta)\chi_{t+1}} \right] \right)^{\frac{\theta-\sigma}{1-\theta}}$$

Meaning that in equilibrium we may express the stochastic discount factor as:

$$M_{t+1} = \beta \left(1 + g_{PC} \right)^{-\sigma} \left(e^{-\theta\chi_{t+1}} \right) \left(E_t\left[e^{(1-\theta)\chi_{t+1}} \right] \right)^{\frac{\theta-\sigma}{1-\theta}}$$

where we have defined growth in output per capita as

$$1 + g_{PC} = \frac{1+g_T}{1+g_L}.$$

E Euler Equation on BGP

Here we show the full derivations of the Euler equation on the risky balanced growth path. Using our expression for M_{t+1} in equilibrium we may write the Euler equation as:

$$
\begin{aligned}
1 &= E_t[R^K_{t+1}M_{t+1}] \\
1 &= E_t\left[\left(R_{t+1} + (1-\delta)\frac{1}{Q_{t+1}}\right)Q_t e^{\chi_{t+1}} \cdot \beta\,(1+g_{PC})^{-\sigma}\left(e^{-\theta\chi_{t+1}}\right)\left(E_t\left[e^{(1-\theta)\chi_{t+1}}\right]\right)^{\frac{\theta-\sigma}{1-\theta}}\right] \\
1 &= E_t\left[\left(R_{t+1}Q_t + (1-\delta)\frac{Q_t}{Q_{t+1}}\right) \cdot \beta\,(1+g_{PC})^{-\sigma}\left(e^{(1-\theta)\chi_{t+1}}\right)\left(E_t\left[e^{(1-\theta)\chi_{t+1}}\right]\right)^{\frac{\theta-\sigma}{1-\theta}}\right] \\
1 &= E_t\left[\left(R_{t+1}Q_t + (1-\delta)\frac{Q_t}{Q_{t+1}}\right) \cdot \beta\,(1+g_{PC})^{-\sigma}\left(E_t\left[e^{(1-\theta)\chi_{t+1}}\right]\right)^{\frac{\theta-\sigma}{1-\theta}+\frac{1-\theta}{1-\theta}}\right] \\
1 &= E_t\left[\left(R_{t+1}Q_t + (1-\delta)\frac{Q_t}{Q_{t+1}}\right)\right]\beta^\star
\end{aligned}
$$

where we've defined the *effective discount factor*:

$$
\beta^\star = \beta\,(1+g_{PC})^{-\sigma}\left(E_t\left[e^{(1-\theta)\chi_{t+1}}\right]\right)^{\frac{1-\sigma}{1-\theta}}
$$

We may use that $R_t = \frac{\alpha}{\mu}\frac{Y_t}{K_t}$ to write:

$$
R_{t+1}Q_t = \frac{\alpha}{\mu}\frac{Z_{t+1}K^{\alpha}_{t+1}(S_{t+1}N_{t+1})^{1-\alpha}}{K_{t+1}}Q_t = \frac{\alpha}{\mu}Z_{t+1}K^{\alpha-1}_{t+1}(S_{t+1}N_{t+1})^{1-\alpha}Q_t
$$

Inserting $K_t = k_t S_t T_t Q_t$

$$
= \frac{\alpha}{\mu}Z_{t+1}(k_{t+1}S_{t+1}T_{t+1}Q_{t+1})^{\alpha-1}(S_{t+1}\bar{N}L_{t+1})^{1-\alpha}Q_t
$$

Inserting $T_{t+1} = L_{t+1}Z^{\frac{1}{1-\alpha}}_{t+1}Q^{\frac{\alpha}{1-\alpha}}_{t+1}$

$$
\begin{aligned}
&= \frac{\alpha}{\mu}Z_{t+1}\left(k_{t+1}\left(L_{t+1}Z^{\frac{1}{1-\alpha}}_{t+1}Q^{\frac{\alpha}{1-\alpha}}_{t+1}\right)Q_{t+1}\right)^{\alpha-1}(\bar{N}L_{t+1})^{1-\alpha}Q_t \\
&= \frac{\alpha}{\mu}k^{\alpha-1}_{t+1}\left(\left(Q^{\frac{\alpha}{1-\alpha}}_{t+1}\right)Q_{t+1}\right)^{\alpha-1}\bar{N}^{1-\alpha}Q_t \\
&= \frac{\alpha}{\mu}k^{\alpha-1}_{t+1}\bar{N}^{1-\alpha}Q^{\frac{\alpha(\alpha-1)}{1-\alpha}}_{t+1}Q^{\alpha-1}_{t+1}Q_t \\
&= \frac{\alpha}{\mu}k^{\alpha-1}_{t+1}\bar{N}^{1-\alpha}Q^{\alpha-1-\alpha}_{t+1}Q_t \\
&= \frac{\alpha}{\mu}(k^\star)^{\alpha-1}\bar{N}^{1-\alpha}\frac{Q_t}{Q_{t+1}} \\
&= \frac{\alpha}{\mu}(k^\star)^{\alpha-1}\bar{N}^{1-\alpha}\frac{1}{1+g_Q}
\end{aligned}
$$

Inserting this[33] result in the Euler Equation yields:

$$1 = E_t \left[\left(\frac{\alpha}{\mu} \left(\frac{k^\star}{\bar{N}} \right)^{\alpha-1} \frac{1}{1+g_Q} + \frac{1-\delta}{1+g_Q} \right) \right] \beta^\star$$
$$1 = \left(\frac{\alpha}{\mu} \left(\frac{k^\star}{\bar{N}} \right)^{\alpha-1} \frac{1}{1+g_Q} + \frac{1-\delta}{1+g_Q} \right) \beta^\star$$

Consequently:

$$\frac{1}{\beta^\star} = \left(\frac{\alpha}{\mu} \left(\frac{k^\star}{\bar{N}} \right)^{\alpha-1} \frac{1}{1+g_Q} + \frac{1-\delta}{1+g_Q} \right)$$

E.1 Rate of return version

We define the composite parameter:

$$\beta^\star = \beta \left(1 + g_{PC}\right)^{-\sigma} \left(E_t \left[e^{(1-\theta)\chi_{t+1}} \right] \right)^{\frac{1-\sigma}{1-\theta}}$$

Using the definitions

$$r^\star = \frac{1}{\beta^\star} - 1 \approx -\ln \beta^\star \qquad\qquad \rho = \frac{1}{\beta} - 1 \approx -\ln \beta$$

we may write the composite parameter in its rate of return version:

$$\beta^\star = \beta \left(1 + g_{PC}\right)^{-\sigma} \left(E_t \left[e^{(1-\theta)\chi_{t+1}} \right] \right)^{\frac{1-\sigma}{1-\theta}}$$
$$\ln \beta^\star = \ln \beta - \sigma \ln \left(1 + g_{PC}\right) + \frac{1-\sigma}{1-\theta} \ln \left(E_t \left[e^{(1-\theta)\chi_{t+1}} \right] \right)$$
$$-\ln \beta^\star = -\ln \beta + \sigma \ln \left(1 + g_{PC}\right) + \frac{\sigma-1}{1-\theta} \ln \left(E_t \left[e^{(1-\theta)\chi_{t+1}} \right] \right)$$
$$r^\star \approx \rho + \sigma g_{PC} + \sigma \frac{1-\frac{1}{\sigma}}{1-\theta} \ln \left(E_t \left[e^{(1-\theta)\chi_{t+1}} \right] \right)$$

[33] Note that Farhi and Gourio (2018) includes $Q^\star = 1$ in

$$R_{t+1} Q_t = \frac{\alpha}{\mu} (k^\star)^{\alpha-1} \bar{N}^{1-\alpha} \frac{Q^\star}{1+g_Q}.$$

This is done to illustrate that we normalise wrt. current costs

F Derivation of Gordon Growth formula

Let P_t denote the price of equity at time t and D_t the dividend payments. Then the (relative) return on equity is given as:

$$R^E_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$$

which we may rearrange as:

$$\begin{aligned}
R^E_{t+1} &= \frac{P_{t+1} + D_{t+1}}{D_t} \cdot \frac{D_t}{P_t} \\
R^E_{t+1} &= \left(\frac{P_{t+1}}{D_t} + \frac{D_{t+1}}{D_t}\right)\frac{D_t}{P_t} \\
R^E_{t+1} &= \left(\frac{P_{t+1}}{D_{t+1}} + 1\right)\frac{D_{t+1}}{D_t}\frac{D_t}{P_t} \qquad \text{(F.1)}
\end{aligned}$$

We restate the Euler equation:

$$1 = E_t\left[M_{t+1}R^E_{t+1}\right]$$

Inserting our expression for return on equity from (F.1) yields:

$$\begin{aligned}
1 &= E_t\left[M_{t+1}\left(\frac{P_{t+1}}{D_{t+1}} + 1\right)\frac{D_{t+1}}{D_t}\frac{D_t}{P_t}\right] \\
\frac{P_t}{D_t} &= E_t\left[M_{t+1}\left(\frac{P_{t+1}}{D_{t+1}} + 1\right)\frac{D_{t+1}}{D_t}\right]
\end{aligned}$$

The price dividend ratio is assumed constant along the risky balanced growth path which allows rearranging the expression as:

$$\frac{P^\star}{D^\star} = \left(\frac{P^\star}{D^\star} + 1\right)E_t\left[M_{t+1}\frac{D_{t+1}}{D_t}\right]$$

We use that $E(e^{\chi_{t+1}}) = 1$, which means that we may rewrite:

$$E_t\left[M_{t+1}\frac{D_{t+1}}{D_t}\right] = E_t\left[M_{t+1}e^{\chi_{t+1}}\frac{D_{t+1}}{D_t}\right]$$

Noting that $\beta^\star = M_{t+1}e^{\chi_{t+1}}$ (eq. (3.23)) we may state the following:

$$= E_t\left[\frac{D_{t+1}}{D_t}\right]\beta^\star$$

We define dividends as $D_t = \Pi_t$ assuming that households own all capital and receive dividends equal to firms' profits. Inserting detrended values we may write

$$\begin{aligned}
&= E_t\left[\frac{\Pi_{t+1}}{\Pi_t}\right]\beta^\star = E_t\left[\frac{s_\Pi Y_{t+1}}{s_\Pi Y_t}\right]\beta^\star \\
&= E_t\left[\frac{s_\Pi y_{t+1}T_{t+1}S_{t+1}}{s_\Pi y_t T_t S_t}\right]\beta^\star
\end{aligned}$$

In equilibrium detrended variables are constant, hence

$$\begin{aligned}
&= E_t\left[\frac{T_{t+1}S_{t+1}}{T_tS_t}\right]\beta^\star = E_t\left[(1+g_T)e^{\chi_{t+1}}\right]\beta^\star \\
&= (1+g_T)\beta^\star.
\end{aligned}$$

Inserting this in the original expression yields:

$$\begin{aligned}
\frac{P^\star}{D^\star} &= \left(\frac{P\star}{D^\star}+1\right)\beta^\star(1+g_T) \\
\beta^\star\frac{1}{(1+g_T)} &= 1+\frac{1}{\frac{P\star}{D^\star}}
\end{aligned}$$

$$\boxed{\frac{P^\star}{D^\star} = \frac{\beta^\star(1+g_T)}{1-\beta^\star(1+g_T)}} \tag{F.2}$$

G Proof that $b_H = b$

Assume that χ_{t+1} follows a three-point distribution given as:

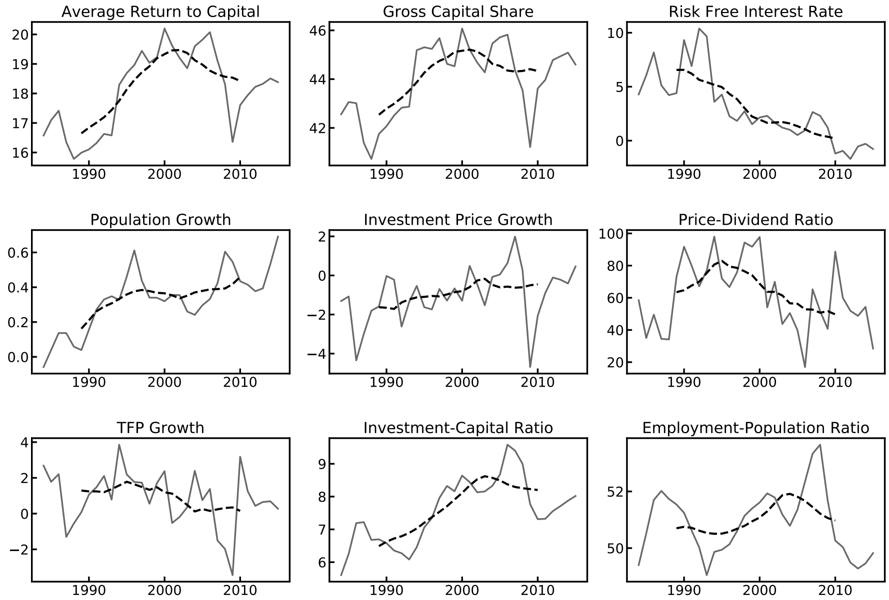
$$\begin{aligned}
\chi_{t+1} &= 0 \text{ with probability } 1-2p \\
\chi_{t+1} &= \ln(1-b) \text{ with probability } p \\
\chi_{t+1} &= \ln(1+b_H) \text{ with probability } p
\end{aligned}$$

If $E(\chi_{t+1}) = 1$ then it follows that:

$$\begin{aligned}
1 &= E(\chi_{t+1}) \\
1 &= (1-2p)e^0 + p\cdot e^{\ln 1-b} + p\cdot e^{\ln 1+b_H} \\
1 &= 1 + p((1-b)+(1+b_H)-2) \\
0 &= p(b_H-b) \\
b_H &= b
\end{aligned}$$

H Analysis for Denmark

Figure 3: Macroeconomic and Financial Trends, Denmark, 1984 – 2015



The nine data series targeted in the estimation, along with their respective 11-year centered moving average (dashed). All series are reported in percentages, except the price dividend ratio.

This part of the appendix presents and comments the results of the empirical analysis for Denmark, while the next section presents the empirical results for a broader range of other countries. We once again note that the results found for Denmark and other small economies can be misleading, as we apply a closed-economy framework.

As evident from Table 7, we find increased savings supply, decreased markups, and increased probability of a shock to the economy. Depreciation, as mentioned and discussed in Section 5, quadruples, and the Cobb-Douglas parameter increases slightly.

The relatively stable income shares observed in the economy leads to the off-setting effect between markups and α, and the large increase in depreciation serves two purposes: It keeps the (gross) average return to capital high, and it dilutes the capital stock to match the observed moments. On all other accounts, the estimated parameter changes are similar to that of Great Britain.

Turning to the decomposition, we once again find that the increase in β decreases the gap between average return to capital and real risk free interest rate. The magnitude is however smaller, as the decrease in markups lowers the average return to capital. The direct effect of decreasing markups is naturally that the gap decreases further, contrary to the case of Great Britain.

If accepting the results for Denmark, an interesting exercise is to subtract the estimated physical depreciation from average gross return to capital. This yields an increase in the spread between return to capital and interest rates of merely 1.25 percentage points, thus not much remains to be explained. Yet, this is a pseudo result and should not be given much weight.

Table 6: Targeted Moments, Denmark

	Averages		
	1984 - 1999	2000 - 2015	Change
Average Return to Capital	17.406	18.758	1.352
Gross Capital Share	43.334	44.585	1.251
Risk Free Interest Rate	5.297	0.657	−4.640
Price-Dividend Ratio	68.676	53.909	−14.767
Investment-Capital Ratio	6.890	8.245	1.354
TFP Growth	1.384	0.381	−1.002
Investment Price Growth	−1.492	−0.491	1.001
Population Growth	0.248	0.411	0.163
Employment-Population Ratio	50.683	51.126	0.442

Table 7: Estimated Parameters, Denmark

		Estimates		
Parameter name	Symbol	1984 - 1999	2000 - 2015	Change
Discount factor	β	0.966	0.978	0.012
Markup	μ	1.288	1.258	−0.030
Disaster probability	p	0.000	0.030	0.029
Depreciation, pct.	δ	1.510	6.252	4.742
Cobb-Douglas parameter	α	0.270	0.303	0.033
Population growth, pct.	g_L	0.248	0.411	0.163
TFP growth, pct.	g_Z	2.186	0.601	−1.585
Technological change, pct.	g_Q	1.492	0.491	−1.001
Labour Supply	$\bar{N}$	0.507	0.511	0.004

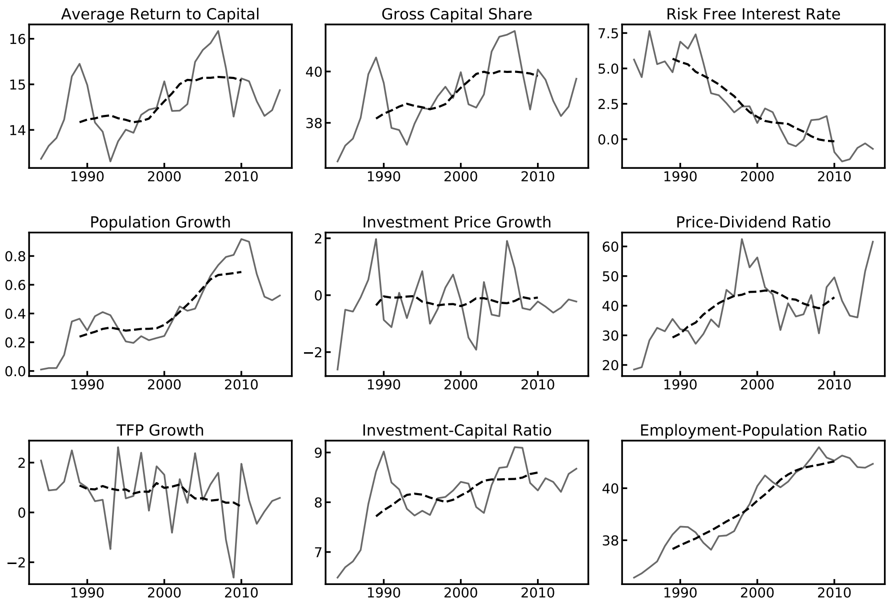
Table 8: Decomposition, Denmark

	Data			*Decomposition of* Δ								
	P1	P2	Δ	β	μ	p	δ	α	g_L	g_Z	g_Q	$\bar{N}$
Average Return to Capital	17.41	18.76	1.35	−2.45	−0.98	1.08	9.28	−0.86	0.00	−2.26	−2.45	0.00
Gross Capital Share	43.33	44.58	1.25	0.00	−1.33	0.00	0.00	2.58	0.00	0.00	0.00	0.00
Risk Free Interest Rate	5.30	0.66	−4.64	−1.22	0.00	−2.16	0.00	0.08	0.00	−1.13	−0.21	0.00
Price-Dividend Ratio	68.68	53.91	−14.77	79.93	0.00	8.90	0.00	−73.58	−51.01	−28.74	49.73	0.00
Investment-Capital Ratio	6.89	8.24	1.35	0.00	0.00	0.00	4.74	0.16	0.17	−2.27	−1.44	0.00
TFP Growth	1.38	0.38	−1.00	0.00	0.04	0.00	0.00	0.00	0.00	−1.26	0.21	0.00
Investment Price Growth	−1.49	−0.49	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
Population Growth	0.25	0.41	0.16	0.00	0.00	0.00	0.00	0.00	0.16	0.00	0.00	0.00
Employment-Population Ratio	50.68	51.13	0.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.44
Spread	12.11	18.10	5.99	−1.23	−0.98	3.24	9.28	−0.94	0.00	−1.13	−2.25	0.00

I Empirical results from other countries

I.1 Belgium

Figure 4: Macroeconomic and Financial Trends, Belgium, 1984 – 2015



The nine data series targeted in the estimation of the model, along with their respective 11-year centered moving average (dashed). All series are reported in percentages, except the price dividend ratio.

Table 9: Estimated Parameters, Belgium

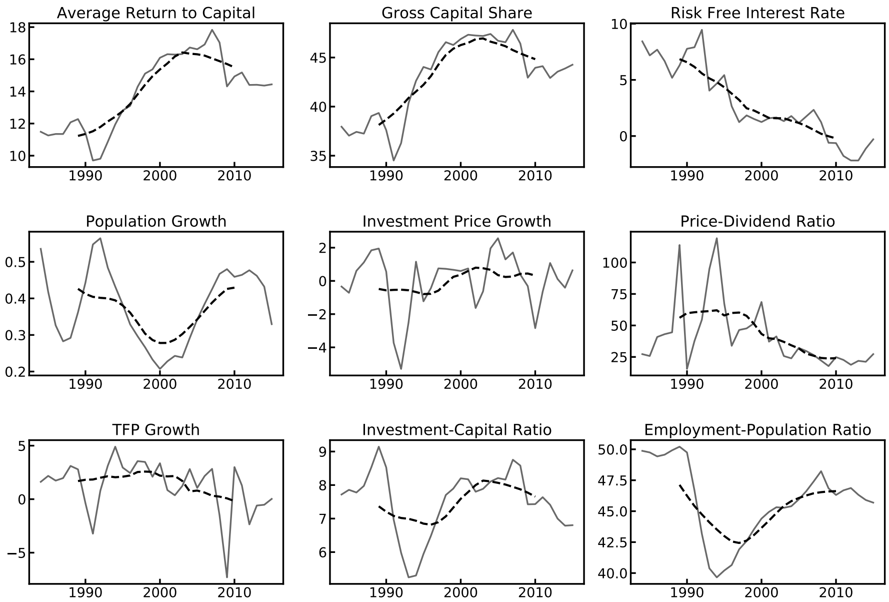
		Estimates		
Parameter name	Symbol	1984 - 1999	2000 - 2015	Change
Discount factor	β	0.961	0.974	0.013
Markup	μ	1.103	1.124	0.021
Disaster probability	p	0.004	0.042	0.038
Depreciation, pct.	δ	5.434	6.613	1.179
Cobb-Douglas parameter	α	0.320	0.322	0.002
Population growth, pct.	g_L	0.232	0.592	0.360
TFP growth, pct.	g_Z	1.219	0.549	-0.670
Technological change, pct.	g_Q	0.224	0.295	0.071
Labour Supply	$\bar{N}$	0.380	0.408	0.028

Table 10: Decomposition, Belgium

	Data			*Decomposition of* Δ								
	P1	P2	Δ	β	μ	p	δ	α	g_L	g_Z	g_Q	$\bar{N}$
Average Return to Capital	14.19	14.99	0.80	−1.85	0.71	0.94	1.60	−0.02	−0.00	−0.69	0.13	0.00
Gross Capital Share	38.40	39.70	1.31	0.00	1.15	0.00	0.00	0.16	0.00	0.00	0.00	0.00
Risk Free Interest Rate	4.67	0.25	−4.42	−1.34	0.00	−2.60	0.00	0.00	0.00	−0.50	0.02	0.00
Price-Dividend Ratio	34.92	43.13	8.21	23.70	0.00	−13.09	0.00	0.04	6.93	−9.71	0.33	0.00
Investment-Capital Ratio	7.81	8.46	0.66	0.00	0.00	0.00	1.18	0.00	0.37	−1.00	0.11	0.00
TFP Growth	1.09	0.46	−0.63	0.00	−0.02	0.00	0.00	0.00	0.00	−0.61	−0.01	0.00
Investment Price Growth	−0.22	−0.30	−0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	−0.07	0.00
Population Growth	0.23	0.59	0.36	0.00	0.00	0.00	0.00	0.00	0.36	0.00	0.00	0.00
Employment-Population Ratio	37.96	40.78	2.82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.82
Spread	9.52	14.74	5.22	−0.51	0.71	3.53	1.60	−0.02	−0.00	−0.19	0.11	0.00

I.2 Finland

Figure 5: Macroeconomic and Financial Trends, Finland, 1984 – 2015



The nine data series targeted in the estimation of the model, along with their respective 11-year centered moving average (dashed). All series are reported in percentages, except the price dividend ratio.

Table 11: Estimated Parameters, Finland

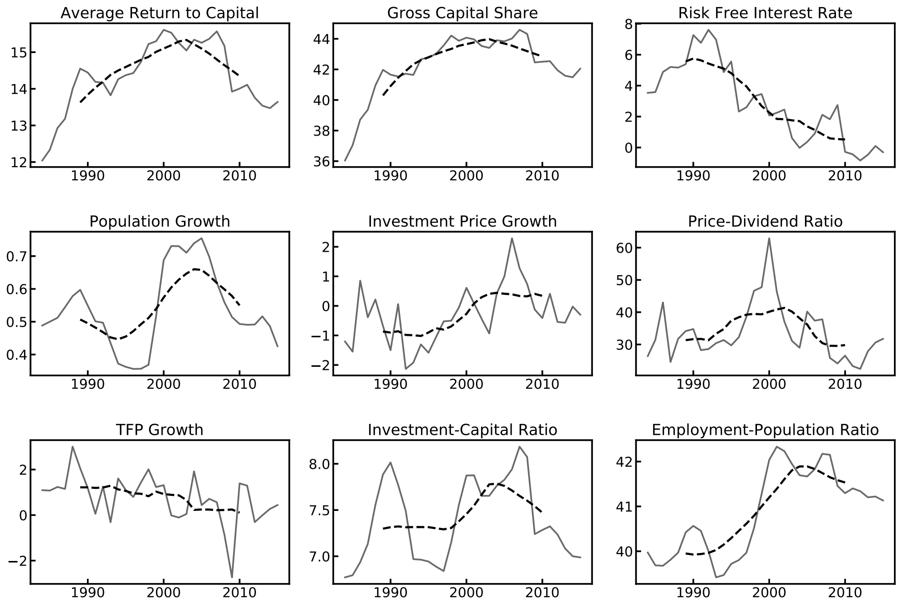
		Estimates		
Parameter name	Symbol	1984 - 1999	2000 - 2015	Change
Discount factor	β	0.961	0.968	0.007
Markup	μ	1.109	1.150	0.041
Disaster probability	p	0.005	0.047	0.043
Depreciation, pct.	δ	2.876	7.166	4.290
Cobb-Douglas parameter	α	0.339	0.374	0.036
Population growth, pct.	g_L	0.387	0.371	−0.017
TFP growth, pct.	g_Z	2.304	0.436	−1.867
Technological change, pct.	g_Q	0.304	−0.288	−0.592
Labour Supply	$\bar{N}$	0.455	0.461	0.007

Table 12: Decomposition, Finland

	Data			*Decomposition of* Δ								
	P1	P2	Δ	β	μ	p	δ	α	g_L	g_Z	g_Q	$\bar{N}$
Average Return to Capital	12.14	15.76	3.62	−1.02	1.18	1.05	5.84	−0.29	0.00	−2.05	−1.08	0.00
Gross Capital Share	40.35	45.57	5.22	0.00	2.07	0.00	0.00	3.15	0.00	0.00	0.00	0.00
Risk Free Interest Rate	5.50	0.33	−5.17	−0.74	0.00	−2.85	0.00	0.06	−0.00	−1.47	−0.17	0.00
Price-Dividend Ratio	54.00	28.84	−25.16	17.00	0.00	−14.49	0.00	1.91	−0.36	−25.70	−3.51	0.00
Investment-Capital Ratio	7.26	7.77	0.51	0.00	0.00	0.00	4.29	0.12	−0.02	−2.94	−0.94	0.00
TFP Growth	2.07	0.42	−1.65	0.00	−0.04	0.00	0.00	0.00	−0.00	−1.67	0.06	0.00
Investment Price Growth	−0.30	0.29	0.59	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.59	0.00
Population Growth	0.39	0.37	−0.02	0.00	0.00	0.00	0.00	0.00	−0.02	0.00	0.00	0.00
Employment-Population Ratio	45.45	46.14	0.68	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.68
Spread	6.64	15.43	8.79	−0.29	1.18	3.90	5.84	−0.35	0.00	−0.58	−0.91	0.00

I.3 France

Figure 6: Macroeconomic and Financial Trends, France, 1984 – 2015



The nine data series targeted in the estimation of the model, along with their respective 11-year centered moving average (dashed). All series are reported in percentages, except the price dividend ratio.

Table 13: Estimated Parameters, France

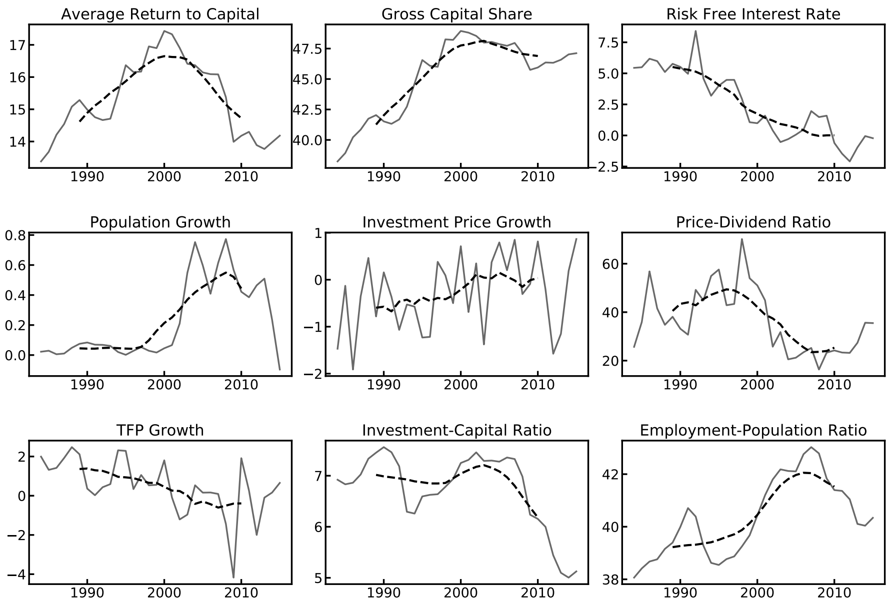
		Estimates		
Parameter name	Symbol	1984 - 1999	2000 - 2015	Change
Discount factor	β	0.956	0.970	0.013
Markup	μ	1.122	1.137	0.015
Disaster probability	p	0.013	0.035	0.021
Depreciation, pct.	δ	3.191	6.864	3.673
Cobb-Douglas parameter	α	0.341	0.354	0.012
Population growth, pct.	g_L	0.471	0.603	0.132
TFP growth, pct.	g_Z	1.482	0.280	-1.202
Technological change, pct.	g_Q	0.830	-0.218	-1.048
Labour Supply	$\bar{N}$	0.400	0.417	0.016

Table 14: Decomposition, France

	Data			*Decomposition of* Δ								
	P1	P2	Δ	β	μ	p	δ	α	g_L	g_Z	g_Q	$\bar{N}$
Average Return to Capital	14.00	14.66	0.66	−2.00	0.44	0.54	5.04	−0.11	0.00	−1.32	−1.91	0.00
Gross Capital Share	41.30	43.14	1.84	0.00	0.75	0.00	0.00	1.10	0.00	0.00	0.00	0.00
Risk Free Interest Rate	4.91	0.82	−4.09	−1.43	0.00	−1.46	0.00	0.02	0.00	−0.94	−0.29	0.00
Price-Dividend Ratio	33.73	33.39	−0.34	19.76	0.00	−5.36	0.00	0.30	1.91	−12.92	−4.02	0.00
Investment-Capital Ratio	7.23	7.56	0.33	0.00	0.00	0.00	3.67	0.04	0.13	−1.87	−1.64	0.00
TFP Growth	1.25	0.27	−0.98	0.00	−0.01	0.00	0.00	0.00	0.00	−1.08	0.11	0.00
Investment Price Growth	−0.83	0.22	1.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.05	0.00
Population Growth	0.47	0.60	0.13	0.00	0.00	0.00	0.00	0.00	0.13	0.00	0.00	0.00
Employment-Population Ratio	40.05	41.69	1.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.65
Spread	9.09	13.85	4.75	−0.58	0.44	1.99	5.04	−0.13	0.00	−0.38	−1.63	0.00

I.4 Italy

Figure 7: Macroeconomic and Financial Trends, Italy, 1984 – 2015



The nine data series targeted in the estimation of the model, along with their respective 11-year centered moving average (dashed). All series are reported in percentages, except the price dividend ratio.

Table 15: Estimated Parameters, Italy

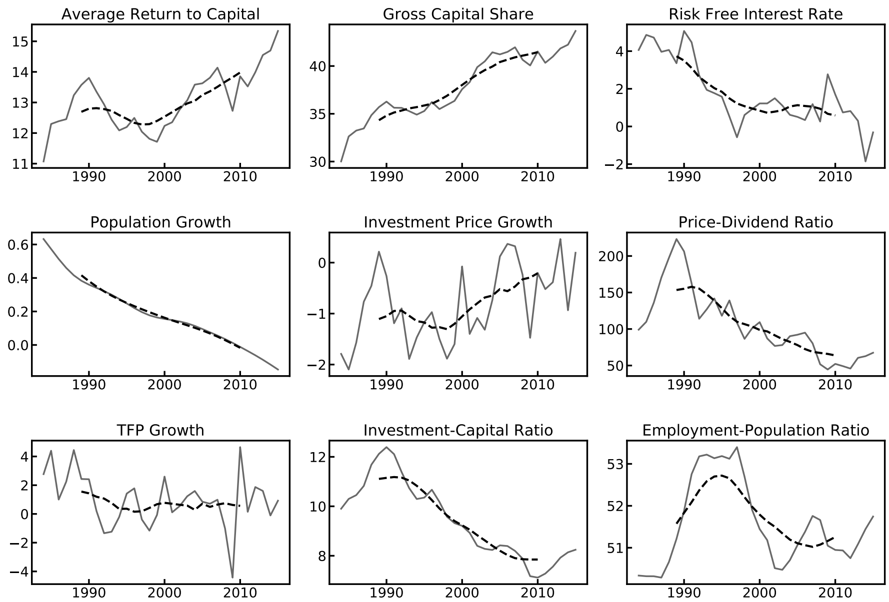
		Estimates		
Parameter name	Symbol	1984 - 1999	2000 - 2015	Change
Discount factor	β	0.965	0.971	0.005
Markup	μ	1.204	1.196	−0.007
Disaster probability	p	0.001	0.037	0.036
Depreciation, pct.	δ	3.711	6.604	2.893
Cobb-Douglas parameter	α	0.315	0.371	0.056
Population growth, pct.	g_L	0.039	0.407	0.368
TFP growth, pct.	g_Z	1.589	−0.312	−1.901
Technological change, pct.	g_Q	0.565	0.016	−0.549
Labour Supply	$\bar{N}$	0.392	0.415	0.024

Table 16: Decomposition, Italy

	Data			*Decomposition of* Δ								
	P1	P2	Δ	β	μ	p	δ	α	g_L	g_Z	g_Q	$\bar{N}$
Average Return to Capital	15.21	15.40	0.19	−0.90	−0.21	1.07	4.59	−0.83	−0.00	−2.39	−1.15	0.00
Gross Capital Share	43.06	47.38	4.32	0.00	−0.33	0.00	0.00	4.65	0.00	0.00	0.00	0.00
Risk Free Interest Rate	4.85	0.14	−4.71	−0.56	0.00	−2.59	0.00	0.06	−0.00	−1.48	−0.15	0.00
Price-Dividend Ratio	44.61	28.30	−16.31	9.35	0.00	−10.08	0.00	1.58	6.25	−21.07	−2.34	0.00
Investment-Capital Ratio	6.92	6.54	−0.38	0.00	0.00	0.00	2.89	0.12	0.37	−2.93	−0.85	0.00
TFP Growth	1.23	−0.26	−1.50	0.00	0.00	0.00	0.00	0.00	0.00	−1.60	0.09	0.00
Investment Price Growth	−0.56	−0.02	0.55	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.55	0.00
Population Growth	0.04	0.41	0.37	0.00	0.00	0.00	0.00	0.00	0.37	0.00	0.00	0.00
Employment-Population Ratio	39.16	41.53	2.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.37
Spread	10.36	15.26	4.90	−0.34	−0.21	3.66	4.59	−0.89	−0.00	−0.91	−1.00	0.00

I.5 Japan

Figure 8: Macroeconomic and Financial Trends, Japan, 1984 – 2015



The nine data series targeted in the estimation of the model, along with their respective 11-year centered moving average (dashed). All series are reported in percentages, except the price dividend ratio.

Table 17: Estimated Parameters, Japan

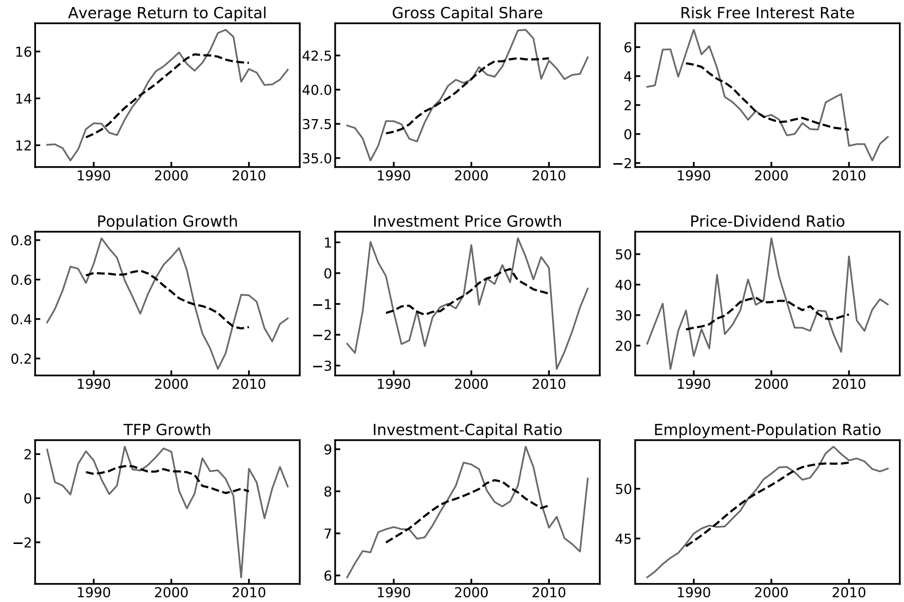
		Estimates		
Parameter name	Symbol	1984 - 1999	2000 - 2015	Change
Discount factor	β	0.979	0.982	0.003
Markup	μ	1.028	1.140	0.112
Disaster probability	p	0.007	0.024	0.017
Depreciation, pct.	δ	6.743	6.021	−0.722
Cobb-Douglas parameter	α	0.330	0.326	−0.004
Population growth, pct.	g_L	0.349	0.031	−0.317
TFP growth, pct.	g_Z	1.220	0.931	−0.289
Technological change, pct.	g_Q	1.207	0.432	−0.775
Labour Supply	$\bar{N}$	0.520	0.511	−0.008

Table 18: Decomposition, Japan

	Data			*Decomposition of* Δ								
	P1	P2	Δ	β	μ	p	δ	α	g_L	g_Z	g_Q	$\bar{N}$
Average Return to Capital	12.49	13.61	1.12	−0.46	3.60	0.40	−0.91	0.02	0.00	−0.28	−1.25	0.00
Gross Capital Share	34.81	40.85	6.05	0.00	6.43	0.00	0.00	−0.38	0.00	0.00	0.00	0.00
Risk Free Interest Rate	2.75	0.76	−2.00	−0.36	0.00	−1.22	0.00	−0.01	0.00	−0.22	−0.19	0.00
Price-Dividend Ratio	140.06	71.46	−68.60	52.35	0.00	−34.96	0.00	−1.26	−36.41	−25.54	−22.79	0.00
Investment-Capital Ratio	10.77	8.09	−2.68	0.00	0.00	0.00	−0.72	−0.02	−0.33	−0.44	−1.18	0.00
TFP Growth	1.17	0.77	−0.40	0.00	−0.18	0.00	0.00	−0.00	−0.00	−0.27	0.06	0.00
Investment Price Growth	−1.21	−0.43	0.77	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.77	0.00
Population Growth	0.35	0.03	−0.32	0.00	0.00	0.00	0.00	0.00	−0.32	0.00	0.00	0.00
Employment-Population Ratio	51.98	51.13	−0.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	−0.84
Spread	9.74	12.85	3.11	−0.10	3.60	1.62	−0.91	0.03	0.00	−0.06	−1.06	0.00

I.6 The Netherlands

Figure 9: Macroeconomic and Financial Trends, The Netherlands, 1984 – 2015



The nine data series targeted in the estimation of the model, along with their respective 11-year centered moving average (dashed). All series are reported in percentages, except the price dividend ratio.

Table 19: Estimated Parameters, The Netherlands

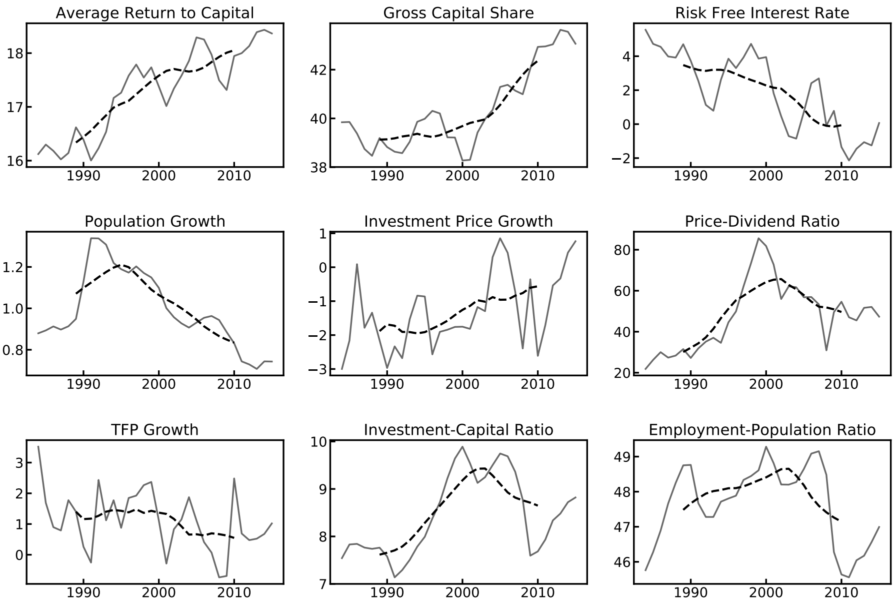
		Estimates		
Parameter name	Symbol	1984 - 1999	2000 - 2015	Change
Discount factor	β	0.952	0.969	0.016
Markup	μ	1.067	1.141	0.074
Disaster probability	p	0.036	0.051	0.015
Depreciation, pct.	δ	2.382	5.740	3.358
Cobb-Douglas parameter	α	0.336	0.338	0.002
Population growth, pct.	g_L	0.599	0.430	-0.170
TFP growth, pct.	g_Z	1.471	0.590	-0.881
Technological change, pct.	g_Q	1.225	0.480	-0.745
Labour Supply	$\bar{N}$	0.457	0.523	0.066

Table 20: Decomposition, The Netherlands

	Data			*Decomposition of* Δ								
	P1	P2	Δ	β	μ	p	δ	α	g_L	g_Z	g_Q	$\bar{N}$
Average Return to Capital	13.04	15.53	2.49	−2.40	2.38	0.34	4.39	−0.01	0.00	−0.92	−1.29	0.00
Gross Capital Share	37.77	41.96	4.19	0.00	4.01	0.00	0.00	0.18	0.00	0.00	0.00	0.00
Risk Free Interest Rate	3.84	0.38	−3.45	−1.75	0.00	−0.84	0.00	0.00	−0.00	−0.67	−0.19	0.00
Price-Dividend Ratio	27.91	32.24	4.33	18.40	0.00	−2.72	0.00	0.05	−1.90	−7.39	−2.11	0.00
Investment-Capital Ratio	7.12	7.80	0.68	0.00	0.00	0.00	3.36	0.01	−0.17	−1.36	−1.15	0.00
TFP Growth	1.32	0.46	−0.86	0.00	−0.11	0.00	0.00	0.00	−0.00	−0.81	0.07	0.00
Investment Price Growth	−1.23	−0.48	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.75	0.00
Population Growth	0.60	0.43	−0.17	0.00	0.00	0.00	0.00	0.00	−0.17	0.00	0.00	0.00
Employment-Population Ratio	45.69	52.33	6.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.64
Spread	9.20	15.14	5.94	−0.65	2.38	1.18	4.39	−0.02	0.00	−0.25	−1.10	0.00

I.7 USA

Figure 10: Macroeconomic and Financial Trends, USA, 1984 – 2015



The nine data series targeted in the estimation of the model, along with their respective 11-year centered moving average (dashed). All series are reported in percentages, except the price dividend ratio.

Table 21: Estimated Parameters, USA

		Estimates		
Parameter name	Symbol	1984 - 1999	2000 - 2015	Change
Discount factor	β	0.955	0.974	0.019
Markup	μ	1.168	1.196	0.029
Disaster probability	p	0.042	0.049	0.006
Depreciation, pct.	δ	1.192	5.591	4.399
Cobb-Douglas parameter	α	0.292	0.299	0.007
Population growth, pct.	g_L	1.104	0.879	-0.225
TFP growth, pct.	g_Z	2.062	0.933	-1.129
Technological change, pct.	g_Q	1.853	0.745	-1.108
Labour Supply	$\bar{N}$	0.477	0.476	-0.001

Table 22: Decomposition, USA

	Data			*Decomposition of* Δ								
	P1	P2	Δ	β	μ	p	δ	α	g_L	g_Z	g_Q	$\bar{N}$
Average Return to Capital	16.73	17.86	1.13	−3.41	1.04	0.18	7.11	−0.13	−0.00	−1.37	−2.29	0.00
Gross Capital Share	39.33	41.40	2.06	0.00	1.45	0.00	0.00	0.61	0.00	0.00	0.00	0.00
Risk Free Interest Rate	3.62	0.25	−3.37	−2.00	0.00	−0.35	0.00	0.02	−0.00	−0.80	−0.23	0.00
Price-Dividend Ratio	40.42	55.01	14.59	80.53	0.00	−5.69	0.00	1.33	−11.96	−37.32	−12.31	0.00
Investment-Capital Ratio	7.98	8.90	0.92	0.00	0.00	0.00	4.40	0.04	−0.23	−1.66	−1.63	0.00
TFP Growth	1.54	0.67	−0.87	0.00	−0.06	0.00	0.00	0.00	−0.00	−0.98	0.16	0.00
Investment Price Growth	−1.85	−0.75	1.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.11	0.00
Population Growth	1.10	0.88	−0.22	0.00	0.00	0.00	0.00	0.00	−0.22	0.00	0.00	0.00
Employment-Population Ratio	47.71	47.59	−0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	−0.12
Spread	13.11	17.61	4.51	−1.40	1.04	0.53	7.11	−0.15	−0.00	−0.56	−2.06	0.00